INDEX NUMBERS

OF

INDUSTRIAL PRODUCTION

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PREFACE

This study has its origin in a request of the Statistical Commission of the United Nations, made at its third session, that the recommendations on the subject of index numbers of industrial production made by the Committee of Statistical Experts of the League of Nations in 1938, should be reviewed and adapted to new conditions, methodology and concepts. This task was carried out by the Statistical Office of the United Nations with the help of an expert consultant, Professor R.G.D. Allen of the London School of Economics.

With the object of improving national index numbers and making them more comparable, the Statistical Commission at its fifth session in May 1950 adopted the following recommendations which are based on the findings of the review:

(i)   Countries compiling index numbers of industrial production are recommended to cover the industries and trades making up the major groups 11 to 51 inclusive of the International Standard Industrial Classification of all Economic Activities. It is desirable that separate indexes of production be prepared for each of these major groups. When this is not practicable or justified by the importance of production in particular major groups, it is recommended that separate indexes be compiled for the following divisions:

> Mining and quarrying
> Manufacturing
> Construction
> Electricity and gas

and within these divisions for such major groups as are important in the countries concerned.

(ii)   If, pending the collection of more comprehensive series, a country is compelled by absence of data to omit important industries from the index, it is recommended that whole major groups be excluded. If the group "construction" is excluded, this should be clearly stated in the heading and footnotes to the published tables. If other major groups are excluded, countries are recommended to consider whether any total or general index should be compiled.

(iii)   Countries are recommended to adopt, for a monthly or quarterly index of production, the base-weighted (Laspeyres) formula in which quantity (quantum) relatives are averaged with fixed weights determined for the base period.

(iv)   Each country should review the weights of its index every five years and, if necessary, adopt a new weight base in the light of recommendations on the general suitability of different years for this purpose. In order to be able to adopt a new weight base, if it is necessary, each country is recommended to plan to take a census of production for 1952 or a proximate year and thereafter to take censuses at five-yearly intervals or more frequently. Recommendations as to a suitable base year will be made at approximately five yearly intervals by the Statistical Commission starting in 1953.

(v)   Whenever a new index is compiled using new weights, it is recommended that index numbers using both the old and the new weights should be

published covering an overlap period of at least twelve months before or after the date when the change is made.

(vi) Each country is recommended to state the weight in the index of each industry or group of industries for which a separate sub-index is shown.

(vii) Each country is recommended to compute the index on the basis of rates of production per working week shown preferably for "months" (that is, twelve times a year), otherwise for quarters of the calendar years. If, in addition, the primary index is adjusted to eliminate the influence of seasonal factors, including holidays, the method of adjustment should be stated.

The Economic and Social Council at its eleventh session in July 1950 approved these recommendations of the Statistical Commission and adopted a resolution in which it recommended:

(i) That the attention of Governments be drawn to the importance of periodic calculation of the volume of industrial production on an internationally comparable basis;

(ii) That Governments at present compiling index numbers of industrial production should review their compilation in consultation with the Statistical Office of the United Nations so as to improve their comparability with those of other countries;

(iii) That other Governments in whose countries industrial production is important should underake to compile such index numbers.

Within the broad lines of these recommendations of the Statistical Commission and the Economic and Social Council, this study deals with the more technical aspects of the problem of compiling index numbers of industrial production.

I. OBJECT AND USES OF INDEX

1.   This study has two main objects. One is to provide guidance on matters both of basic concepts and of statistical practice to individual countries compiling, or planning to compile, index numbers of production. The other and equally important object is to assist in securing international comparability between national index numbers. Differences in practice will always exist but as these differences are reduced it becomes more feasible to compare and aggregate the several index numbers and hence to strengthen the procedure of international economic analysis.

2.   The general aim of an index number of industrial production is to indicate changes over time in the volume of production of non-agricultural commodities. The index numbers compress many facts into a few simple figures and, in conjunction with other data, their use in economic analysis is in summarizing past developments, forecasting future trends and making decisions on policy.

3.   In micro-economic analysis, an index number of production shown with an industrial grouping enables comparisons to be made of changes in the output of different industries, as between themselves and in relation to such other data on separate industries as employment, wages and earnings. A specific example is the analysis of relative changes in productivity, measured statistically as output per man or per man-hour. In macro-economics, the index serves to assess the significance for the economy as a whole of changes in the volume of industrial output in relation to corresponding changes in population, national income, foreign trade, prices and other aggregates. Indeed, the index has a particular importance in any analysis of economic changes since industrial production is one of the more dynamic and fluctuating elements in the economy. The broad industrial groupings of the index are relevant here and it would be even more useful if a separation could be made, e.g., between production of capital goods and of consumption goods (but on this point see section III below).

4.   For such purposes as these an index of production is needed by all countries with important industries. It is also important that the index numbers of the several countries should be on a comparable basis. They can then be compared one with another and aggregated to show changes in total production, and in the main industrial groups, both for important regions and for the world as a whole. Comparisons would again be made with similar aggregates of population, national income etc.

5.   The title "index number of industrial production" cannot be accepted without question, though it is in general use. Three issues should be raised, relating to the meaning to be attached to the terms "production", "industrial" and "index number"; these can be taken in turn. The result of activity in the whole economy is summarized in value terms in the concept of the national product. The basic concept of "production" in the index of industrial production is the same as in the national product, but in volume or quantum terms and not in value. The concept is the work done, or quantum of output added, in various sectors of industry. This is distinguished from input on the one hand and from deliveries and consumption on the other. It differs from input because it refers not to activity as such but to the results of activity. It differs from deliveries because it comprises work done at all stages of production, including work done on products not yet completed. It differs from consumption also because it does not take account of the terms on which exports are traded for imports. The standard of living may be rising even when domestic output (as measured by the index of production) is falling if there is a favourable turn in the terms of external trade. The relation of the index to the concept of national product is elaborated in section II below.

6.   In its commonly understood form, the index of production does not comprise all activities in the economy. It is convenient and useful to limit its scope to "industry " in the narrow sense of production of commodities, excluding agriculture and services. The components from which it is constructed are the products of industrial establishments and these comprise a very substantial part of the whole economy. Moreover, the products aggregated in the index are generally the more readily measurable elements of the economy. The scope of the index is described more fully in section III below.

7.   No direct aggregation of work done is possible, at least for general use. A money valuation is ruled out because of price movements and a summation in common physical units such as tons is not suitable, except for special purposes. The problem is one of index numbers, in this case of combining a diversity of measures of changing product in various sectors of industry. An index of production is subject to the familiar limitations of all index numbers. The series available are generally an imperfect and incomplete representation of the whole field. Further, given the series, they can be combined according to different formulae which give different results. Finally, given one form of the index (e.g., with fixed base and set of weights), its use is valid only for relatively short-run comparisons. It follows that, like all index numbers, an index of production is only useful if it is available promptly and regularly (preferably monthly) for short-run comparisons and that it is only valid for longer-run comparisons if, at intervals, it is reviewed, revised and related to bench-mark data derived from extensive information, e.g., obtained at a census of production.

8.   In combining the measures of changing product, some gauge must be chosen of the relative importance of the various sectors of industry. For the general purposes set out in paragraphs 3 and 4 above, this gauge will be in value terms. For special purposes, however, index numbers of production based on quite a different gauge of importance could be constructed. Two examples maybe quoted. For some purposes, as during a period of emergency when transport is a limiting factor in the economy, the shipping volume or weight of production is an important measure. The whole of industrial

production, as prepared for transport, can be expressed in terms of the common unit of shipping tons and the index computed is a comparison of total shipping tonnage in different periods. This is equivalent to a valuation and aggregation of production by tonnage per unit of product. Again, for some comparisons of productivity of labour, in which allowance is made for shifting activity between industries, it may be appropriate to weight quantity relatives with man-hours worked. This amounts to a valuation and aggregation of production by use of man-hours per unit of product instead of prices. This problem is examined in some detail in appendix D (paragraphs 6 and 7). The main point, however, is that these are special-purpose index numbers which do not affect the prime need for a general-purpose index.

9.   In the generally accepted title of the index, therefore, the word "production" is perhaps not fully satisfactory for the concept of work done which a general-purpose index requires (paragraph 5). However, it suggests the link with national product, and it is to be preferred to «activity» since the index refers to the results of activity (use of resources) rather than to activity itself. The word «industrial» may sometimes have a wider connotation than is proposed here (paragraph 6) but it is not likely to be misleading in practice.

10.   It is concluded that the title "index number of industrial production" can be retained as the description of an index which combines series representing changes in the volume of work done in various sectors of industry, limited to the production of commodities, excluding agriculture and services.

II. RELATION TO NATIONAL INCOME ACCOUNTING

11.   Activity in the whole economy of a country can be valued in three ways to give, after suitable adjustments, three identical totals: national income, national expenditure and national product. This is essentially a money valuation at current prices. However, there   corresponds to each total a "real" or volume concept with price changes eliminated, which can be approximated by a valuation at constant prices. An index of production relates to the volume concept, i.e., to national income, expenditure and product at constant prices. Moreover, each total can be measured at factor cost or at market prices and it is the former which is relevant here since the index should not be influenced by

the levels of indirect taxation and subsidies. Finally, each total can be taken gross or net, i.e., including or excluding depreciation and similar allowances.

Either gross or net totals can be used and each has its uses in national income accounting. A choice between them can only be made on practical grounds (see paragraph 16 below).

12.   National income is the aggregation of incomes earned by the factors of production, i.e. incomes generated in various sectors of the economy. This approaches the total from the side of inputs and: it is therefore conceptually unsuitable as a basis for an index of production. There are, in any case, difficulties in computing movements in the "volume" of the various components of national income. Compilations of national income are of little assistance in the construction of an index of production.

13.   National expenditure is the aggregation of expenditures on goods and services by consumers, by government and in capital formation. This approaches the total from the side of consumption rather than production. The main item, which can be expressed at constant as well as at current prices, is a valuation of deliveries of finished goods and of services for consumption and for capital formation. Two other items are added to make up the total; the value of the change in stocks and work in progress at all stages of production, and the international trade balance (exports less imports) in goods and services. The main item comprises only finished goods, includes imports and excludes exports. The other two items make the adjustment to domestic activity by allowing for intermediate goods and for external trade.

14.   An index number can be devised to show changes in the volume of deliveries of final goods and services for consumption and capital formation. This corresponds to the main item of national expenditure and extends this item over time in volume terms. In its construction, deliveries could be priced at constant prices, or quantity relatives (from deliveries) could be combined with gross value weights. The index would be composed of commodity series, it would be classified by commodities and it could be related to commodity prices. Though a useful index for many purposes, it does not correspond to an index of production. The coverages are not the same and, to make them agree, the two adjusting items (for stocks and work in progress and for external trade) need to be included in quantum terms. While these may be possible in total, it is not practicable in detail; the index of deliveries is on a commodity basis but the index of production is to be classified by industries. Also, while it may be possible annually, it is very unlikely that the necessary data will be available quarterly or monthly. National expenditure, therefore, can provide no more than an indirect approach to an index of production.

15.   National product is the aggregation of product in the sense of value added, or value of work done, in the various sectors of the economy. This is a total derived from the side of production, a direct approach to the problem of defining an index of production. It would cover industries producing raw materials and semi-finished goods as well as final goods. The index of production can then be regarded as extending the value of national product over time in volume terms, i.e., as a valuation of national product at constant prices. In weighted average form, the index would combine series of the volume of work done in the various industries with weights set by the value of their net outputs (contributions to national product). The index would relate to the output of establishments and would be grouped according to an industrial classification of establishments. This approach is the most direct and the most familiar; it can be adopted as the practical basis in constructing an index of production. This discussion has been in terms of a comprehensive index relating to total production activity. Within this total, an index of industrial production is limited to work done in the industrial sectors of the economy and relates therefore to a similarly limited part of national product.

16.   An index of industrial production, therefore, uses series of volume of work done to extend value of net output calculated as the contributions of the various industries to national product. Two different index numbers arise accordingly as national product is taken gross or net. There is little to choose between them conceptually and both could be constructed for diverse purposes. If a choice has to be made, it must be on practical grounds. One consideration is that, in defining net outputs as weights, the deduction of depreciation allowances can generally be made, particularly in industry detail, only on the basis of accounting for internal revenue purposes; this is both rather arbitrary and also variable in one country to another. Another consideration is that if the net concept is adopted for weights, then the series used to extend net output should also allow for depreciation; this is generally not practicable. The balance, on practical considerations, is in favour of the gross concept with weights equal to net output, before deduction of depreciation allowances, as contributions to gross national product. However, countries which prefer to use weights proportional to contribution to net national product and which have sufficient data to enable them to construct a satisfactory index on this basis should feel free to do so if their particular needs would thereby be served more effectively.

17.   It is concluded that the index of industrial production should be related to the concept of national product (gross or net) at factor cost, that the weights of the index should be values of net output in the sense of contributions to national product (gross or net) and that, because of the practical difficulties in allowing for depreciation, the concept of gross national product is generally to be preferred with weights proportional to net output before deduction of depreciation allowances. However, countries which prefer to use weights proportional to contribution to net national product should feel free to do so if their particular needs would thereby be served more effectively.

III. SCOPE AND GROUPING

18.   As noted in paragraph 6, the scope of the index of production is commonly understood to be limited to the production of non-agricultural commodities. The scope

needs more precise definition with reference to the practicability as well as the desirability of including various activities. Further, the grouping of the index to be based on an industrial classification, i.e. a classification of establishments by types of activity. The appropriate classification is the International Standard Industrial Classification of all Economic Activities (Statistical Office of the United Nations, Statistical Papers, Series M, No. 4). This can be taken as the frame for the definition of the scope and grouping of the index.

19.   Throughout this paper the following terminology has been adopted to describe the groupings of industries falling within the scope of the index of industrial production. The terms "division" and "major group" are used in the same sense that they are employed in the International Standard Industrial Classification, i.e. 1-digit and 2-digit classes. The term "census industry" is used to refer to 3-digit groups in the standard classification or to sub-divisions of those groups for which data on value added can be obtained from a census of industry.

20.   The general scope of the index of industrial production is defined by divisions 1 to 5 inclusive of the Standard Classification. Water and sanitary services (major group 52 of division 5) involve a considerable non-industrial element and should be omitted. All the other activities-mining, manufacturing, construction, electricity and gas-appear to belong to the scope of the index. There can be little, if any disagreement about mining and manufacturing. Construction is a rather more difficult case, but the difficulties arise because the problems of measurement are particularly troublesome in this field. Construction is an important industry, however, and it is desirable that efforts should be made to overcome the practical difficulties in the way of its inclusion. Electricity and manufactured gas raise other difficulties, of a more conceptual kind. Some of the activities included are regarded as non-industrial. This may be true of hydroelectric generation, which is characterized by heavy capitalization, low employment and practically no materials cost. It is certainly true of the distribution of electricity and manufactured gas which cannot be separated, in practice, from production. Nevertheless, the production and distribution of electricity and manufactured gas are sufficiently similar to extraction and manufacturing processes to warrant inclusion. The general scope of the index, therefore, is major groups 11 to 51 inclusive in the International Standard Industrial Classification.

21.   The Standard Classification is one of all economic activities and within major groups 11 to 51 there are several different types of industrial organization including factories (and similar organizations such as mines and generating stations), workshops, handicrafts and work in the home or farm; and within each type of organization, establishments of all sizes. The types included in the "commonly understood" meaning of industry vary considerably from country to country and it is desirable to adopt a standard coverage to avoid international incomparability. The decision what to include can only be taken on practical grounds. It is tempting to define "industry" in terms of establishments of a given type or size, for instance by excluding handicrafts or small establishments, but since the importance of establishments of given types and sizes varies greatly from country to country, the effect of such a procedure would be to lessen comparability, not to improve it. What is necessary is that the great bulk of a country's industrial production in whatever form it is organized, should be covered. Work in the home or farm is excluded because a separate location appears to be necessary before activity can be considered «industry». Apart from the exclusion of this type of activity, it is suggested that the concept of industrial production should be broad, embracing factories, workshops and handicraft establishments of all sizes and that in principle the index should relate to this coverage.

22.   In practice data cannot be expected regularly from the whole of this field. Data can hardly be expected at all from very small establishments since these are, seldom covered even by a detailed census of production. It is suggested, however, that efforts should be made to ensure that the establishments excluded in this way do not account for more than 2 or 3 per cent of the total net output defined as census value added or preferably as contribution to gross or net national product. Therefore, an index computed for census years and weights for any index should be based on virtually complete coverage of the whole field. If a census is taken annually, such an index can be computed on an annual basis. If censuses are taken less frequently, countries should consider the possibility of computing an annual index for periods between census dates covering production of large establishments and a separate index, computed at census intervals, in which the production of large and small establishments is shown separately (see paragraph 108). It is suggested that for this purpose small establishments should be defined in each country so that they do not account for more than 10 percent of the country's net industrial output as recorded at the latest census of production and defined as stated above, leaving 90 per cent to be covered by the annual index. If this is done, the monthly or quarterly index can then be constructed on the basis of a less comprehensive direct coverage and linked to the annual index. It is recommended that the establishments included in the monthly or quarterly index by means of direct returns should account for two-thirds or more of the net output of large establishments, i.e., at least 60 per cent of total net output. The remainder can be included in the monthly or quarterly index through the use of various methods of imputation which are discussed in section VII

23.   Another problem is presented by repair work. At one end of the scale, repair work is performed under non-industrial conditions (e.g. in connexion with a service trade, as boot and shoe repair) while, at the other extreme, it involves the same kind of activity as new production (e.g. repairing of ships and locomotives, building repairs and demolition). The best solution seems to be to exclude repair work generally, unless it is carried on essentially under industrial conditions. Ship and locomotive repairs would be included.

The chief uncertainties are likely to arise on building repairs and motor car repairs; as a broad guide, it is suggested that the first be included and the second excluded. Another service which can be excluded is prospecting for minerals, which appears under mining in the Standard Classification.

24.   For international comparability, it is important that there shall be a common grouping as well as uniform coverage. The most satisfactory arrangement would be that each country should show separate group index numbers for each of the 27 major groups of the Standard Classification included in the coverage. (A list of these major groups is given in appendix B). In view of the very great variations in the importance of different industries in different countries, however, it is not to be expected that all countries will attempt such an analysis. It is recommended, therefore, that each country should compute separate indexes for the four divisions, mining and quarrying, manufacturing, construction, and electricity and gas, and within these divisions for all industries which are important in the country concerned. The industries should be selected so as to correspond to major (2-digit) groups in the Standard Classification, or finer (3-digit) groups which add up to the major groups. For example, coal mining (group 11) and petroleum production (group 13) could be shown. Or, transport equipment (group 38) could be distinguished, together with ship-building and repair (group 381) and other transport equipment, if ship-building is an important activity. In defining industries considerable effort should be made to make the agreement with the Standard Classification reasonably close. However, this should not be pushed too far, since it is wasteful to spend much time on separating out small items for insertion in appropriate broad categories. An example is baby carriages (perambulators) which could appear, in the statistics of various countries, under metal products, transport equipment or miscellaneous manufacturing.

25.   Some broader and very different groupings of industries are often prepared for special purposes. The most important are two-way groupings into capital and consumption goods industries and into durable and non-durable goods industries, but there are others depending on distinctions, for example, between domestic and export goods, or between publicly-produced and privately-produced goods. There are always difficulties with such groupings since they generally depend on the uses of commodities which are often not clear-cut. An industrial grouping of an index of production does not lend itself to distinctions according to commodity use. Groupings of the kind instanced

here are of great use in economic analysis, however, and countries are to be encouraged to experiment with them in their index numbers of production, even if they are rather rough and ready. No proposals for international standardization, however, would seem to be possible at present.

26.   A final problem concerns what should be done when data available for the compilation of a major group index are deemed inadequate for the purpose. The main decision is whether the weight for this group should be imputed to that of another group (or spread between two or three groups), or whether the group should be excluded from the coverage of the index. This is a problem of imputation of the kind discussed more fully in section VII below. If it is decided to impute the group's weight to another group, then the coverage of the index is complete and a total index can be compiled. On the other hand, if the decision is to omit the group, then the question arises whether a total or general index should be compiled at all and, if so, how it should be described. The omission may be on the grounds that the quantum series for the group follows the average movement of all other groups, i.e., the weight of the group is imputed to the total index. The compilation of the index then proceeds with the group omitted. Conceptually, however, the coverage of the index is complete and a total index can be compiled. This procedure should be followed only if there are very good reasons to expect the group to follow the general average. Otherwise the group should be omitted from the coverage; any general index compiled does not represent the whole field. However, if satisfactory data are available for the compilation of indexes for some of the 3-digit groups of which the major group is composed, these may be included in the general index if one is compiled. Construction is a group which may be expected to cause difficulties in practice. The general aim must be to include the group in the index. As an interim measure, pending the collection of data from which an index for construction can be compiled, the whole group can be omitted. It is not desirable to include an index of partial coverage, e.g., residential construction only. If construction is the only group omitted, then a general index maybe compiled provided that it is clearly described as excluding construction. If any other group that is important in the country's economy is omitted, e.g., mining or metal products, the advisability of compiling a general index should be carefully considered. Such an index is very likely to be understood as applying to the whole of industrial production, in spite of clear and repeated statements of the actual coverage. If there is reason to think that production in the omitted section is substantially different from that in the included section, the dangers of misinterpretation are great and no general index should be published. If, in spite of this danger, a general index based on incomplete coverage is published, the group or groups omitted should of course be clearly indicated.

27.   It is concluded that countries compiling index numbers of industrial production should cover the industries and trades making up the major groups 11 to 51 inclusive of the International Standard Industrial Classification of all Economic Activities, i.e., mining and quarrying, manufacturing, construction, and electricity and gas. In principle the index should embrace factory production, workshops and handicrafts, but should exclude work in the home or farm and repair work generally performed in connexion with a service trade. Because of the difficulty of obtaining data from small establishments except at censuses of production, countries should consider compiling an intercensal index covering large establishments, and an index covering the whole field, with production by small establishments shown separately, at census dates. "Small" establishments should be defined in the circumstances of each country, to cover not more than 10 per cent of the country's industrial production. Separate indexes should be compiled for the four divisions mentioned above, and within these divisions for all industries (defined so as to correspond to major groups or industries in the Standard Classification) which are important in the country concerned. As an interim measure, pending the collection of adequate data, major groups or 3-digit groups may be omitted from the coverage. If construction is the only group, omitted, a general index (described as excluding construction) can be compiled. If any other important group is omitted a general index should be published only if the compilers are prepared to have it interpreted as an index of total industrial production, and the group or groups omitted should be clearly indicated.

IV.   FORMULA AND BASE

28.   Many formulae have been used, and more suggested, for the compilation of index numbers; some of them are very complicated in form and require extensive data in compilation. A good deal of attention has been given to the definition of an "ideal" index number. This is a matter which may be usefully pursued when the problem is to compile index numbers at intervals (e.g. annually) with the aid of detailed and extensive data. It is not practicable to use a complicated formula for an index of production which has to be computed promptly, frequently and regularly from data of the kind readily available monthly and quarterly. What is then needed is some simple formula which has a clear interpretation, which is easily computed and which makes use only of data likely to be available promptly and regularly. Moreover, for purposes of international comparison and aggregation, it is important to have a single formula adopted for the national index num-bers. Particular refinements of formulae may be used by those countries fortunate enough to have the necessary data regularly to hand; but these should be in addition to (and not instead of) the compilation of a simple, single form which can be adopted by all.

29.   There can be little doubt about the nature of the formula to be proposed for index numbers of industrial production to be computed regularly (preferably monthly) on an internationally comparable basis. The formula must be one of the well-known aggregate forms in which quantities are valued and aggregated at constant prices. Each of these formulae can be written in equivalent form as a weighted arithmetic mean of quantity relatives. The general form can be written:

$$Q_{01} = \frac{\Sigma pq_1}{\Sigma pq_0} = \frac{\Sigma pq_0(\frac{q_1}{q_0})}{\Sigma pq_0} = \frac{1}{\frac{\Sigma pq_1(\frac{q_0}{q_1})}{\Sigma pq_1}}$$

where suffixes o and 1 indicate the two periods compared, where φ extends over all the items included in the index and where p represents some set of "prices" selected for purposes of making comparison between the "quantities" q_0 and q_1. It remains to select the price set p. Two obvious selections are prices in the base period (po) and prices in the current period (p_1). These give rise to the familiar base-weighted (Laspeyres) form:

$$Q_{01} = \frac{\Sigma p_0 q_1}{\Sigma p_0 q_0} = \frac{\Sigma p_0 q_0(\frac{q_1}{q_o})}{\Sigma p_0 q_0}$$

and current-weighted (Paasche) form:

$$Q'_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_1 q_0} = \frac{1}{\frac{\Sigma p_1 q_1(\frac{q_0}{q_1})}{\Sigma p_1 q_1}}$$

30.   One general property which the volume or quantum index should possess is that if multiplied by a price index it should yield an index of change in value. The change in value of total output from period 0 to period 1, Vol, needs to be split into a price component P_{01} and a volume component Q_{01}

$$V_{01} = Q_{01} \times P_{01}$$

This can never be done uniquely; there is always a choice of components. For example, Q_{01} can be obtained either as a base-weighted or a current-weighted index; the Laspeyres Q_{01}) and Paasche (Q'_{01}) forms given in the previous paragraph are the arithmetic versions of these volume indexes. Similar forms (P_{01} and P'_{01}) can be developed for the price component. The volume index and the price index can be matched off, the base-weighted form of one index with the current-weighted form of the other. Thus the base-weighted form Q_{01} from paragraph 29, when multiplied by the corresponding current-weighted price index P'_{01}, or the current-weighted form Q'_{01} multiplied by the base-weighted P_{01} Yield the value index.

31.   It is necessary to be a little more precise about the "prices" p and "quantities" q in paragraph 29. The concept of the index of industrial production, as developed here, requires that q should be a series representing the volume of work done in a particular industry. Hence p should not be the gross price of the product of the industry; it should be the net price or margin added by the industry and expressed per unit of work done (q). In fact p is better considered and estimated, not by itself, but in conjunction with q (see paragraph 51). The product pq represents some valuation of net output; for example $p_0 q_0$ is value of net output in the base period o and $p_1 q_1$ is value of net output in the current period 1. Hence the weighted average form of Q_{01} or Q'_{01} is preferable for practical purposes to the equivalent aggregate form. Q_{01} is the weighted average of relatives formed from series representing work done (period 1 being compared with period o), weights being values of net output in the base period o. The reciprocal of Q'_{01} is similar but worked backwards from period 1 to period o; the relatives compare period o with period 1 and the weights are values of net output in period 1.

32.   The choice is between Q_{01} and Q'_{01}. The best solution would be to calculate both of these index numbers. For the difference between them is of importance in assessing the significance to be attached to any index of industrial production between the two periods compared. If Q_{01} and Q'_{01} are close together, either can serve as a good indicator. If they are far apart, then there is less significance to be attached to any indicator of change between the two periods. For a regular (e.g. monthly) series of index numbers, however, it is not practicable to compute both Q_{01} and Q'_{01}. A choice must be made and it must clearly be for the base-weighted (Laspeyres) form Q_{01}. This is inevitable since the weights, values of net output, are generally available only for particular periods and only after some time lag. The proposal, then, is that the base-weighted (Laspeyres) form should be used for a regular monthly or quarterly series of index numbers of industrial production.

33.   A form with fixed (base) weights is the only practicable one when current in-formation on weights is not available. Such an index is clearly valid only for short-run comparisons; but this is true of any index number of a given type, of Q'_{01} and of "cross" forms as well as of Q_{01}. One form Q_{01} provides good comparisons between the "base" period o and any other period not far removed; the other form Q'_{01} is good for comparisons between the "current" period 1 and any near period. Neither is necessarily valid for comparing two periods far apart. The index in form Q_{01}, as in any form, needs to be tested, reviewed and (when necessary) re-weighted from time to time. Opportunity should be taken, at times of revision, and more frequently if possible, to compute both Q_{01} and Q'_{01} for the pair of periods under review. The relation between them is always of interest and it is an essential part of the process of revising the index. Over the long period, therefore, it is to be expected that there will be several sets of index numbers, each of base-weighted. form but with changing base and weights, and each suitable for short-run comparisons. There is still the problem of setting up bench-marks at intervals to link together the index numbers into a continuous chain. It is here that proposals on the use of "cross" or "ideal" forms of the index are relevant and need careful examination. The problem is considered in section IX below.

34.   The <u>weight base</u> of the index is the period to which the weights relate. A period of less than a year (e.g. a month or a quarter) is not generally broad enough for the base. On the other hand, an average of a series of years is not often possible as the base period of an index of production since data for weights are usually available only for single years and at intervals. It is suggested, therefore, that a single year be selected for the weight base of the index.   However, countries which take an annual census of industry may use a three-year weight base centred on the year selected by the Statistical Commission as the standard weight base (see paragraph 35) if they prefer to do so. As indicated in paragraph 33, the weight base needs to be reviewed and (when necessary) the weights need to be changed, from time to time.   A review at regular, rather than irregular, intervals is clearly desirable. As a practical rule, it is suggested that a five-yearly review is sufficient. At each such review, a decision is to be taken whether to retain the existing weight base or whether to adopt new weights and, in the latter case, which year to select for the new weight base. There must be some flexibility, if an entirely unsuitable year is to be avoided, but the choice of new base can be limited to the period one year on either side of the year of review. To maintain as much continuity as possible, whenever a change of weight base is made, the index should be compiled on the old and on the new base for an overlap period of twelve months.

35.   The interests of international comparability need to be considered in proposals on the change of weight base in national index numbers. It is highly desirable, if index numbers are to be compared and aggregated into regional and world totals, that all countries synchronize the changes of weight base in their index numbers; they should simultaneously adopt new weights and the weights should all relate to the same year. Desirable as this is, it is not practicable to expect that any hard and fast rules can be followed. To achieve an essential minimum of comparability, it is suggested that all countries should agree to review their index numbers at or about the same time and to repeat the review every five years. At the time of each common review, the Statistical Commission of the United Nations should be asked to make a definite recommendation on change of weight base for individual countries to follow, i.e., whether or not a new weight base should be adopted by all countries and, if so, which particular year (around the date of review) should be selected as the new base.

36.   The periodic reviews of the index, and particularly any change of weight base, are dependent on the collection of comprehensive data on production, by quantity and price. Such data are generally to be obtained only in censuses or sample surveys of the whole field. The proposals of the previous paragraph can only be carried out if all countries plan to take a census of production, or a wide sample survey, every five yeairs and in or around same year on every occasion. It is possible to have small differences in timing of censuses without giving up concerted changes in weight base. The weights for a base year adopted by international agreement can be estimated by extrapolation from data obtained at a census or survey in a different year, provided that the base and census years are close together. Since weights are relative (and not absolute) values of net output, all that is required is that the extrapolation should be free of bias. It could often be made, for example, by using such partial data as the wage bill by industries.

37.   The <u>comparison base</u> of the index is the year or period written as 100 in the series of index numbers. For the index of an individual country, it is generally preferable that the

comparison base should be the same year as the weight base, if only to avoid misunderstanding about which year is used as the basis of weighting. However, if the country finds it convenient, a comparison base different from the weight base can be selected. This does not imply any change in the form or weighting of the index. Given the series on the weight base as 100, all that is done is to divide through by the value of the index in the selected comparison base, i.e. the whole series is re-scaled in proportion so that the index in the comparison base appears at 100. However, the series on the comparison base as 100 can be computed in practice from quantity relatives on the comparison base, without reference to the weight base. If this is done, the weights for the quantity relatives are not values of net output at the weight base, but those values adjusted for quantity changes from the weight base to the comparison base. This is shown in appendix D (paragraph 8).

38.   It is concluded that a general-purpose index of production compiled regularly (preferably monthly) should be of the base-weighted (Laspeyres) form in which quantum relatives are averaged with values of net output in the base-period. The index should be tested, reviewed and (when necessary) re-weighted from time to time, say at five-yearly intervals. The Statistical Commission of the United Nations should make recommendations every five years with the object of synchronizing the change of weight base in national index numbers. All countries should plan to take a census of production, or a wide sample survey, every five years, synchronized with each other and with the times of review of the index. The comparison base of the index is preferably, but not necessarily, to be the same as the weight base.

V. WEIGHTS AND SERIES: WEIGHTS

39.   The main task in the construction of an index of industrial production is to determine the weights to be attached to the various industries in the base period and to select the particular series of quantum data to measure current changes. The choice of weights and series must be considered together, since they are interrelated, but it is convenient, in practice, to proceed in two steps. The first stage is to fix the main industry groups within the general industrial classification and the weights appropriate to them. The second stage is to determine the particular series to represent each main industry group and the internal weighting, i.e., the way the series are combined within the main industry group.

40.   The main industry grouping and the weights are generally to be determined from data on net output supplied in a census of production. In practice, however, censuses which give values of net output give them only down to a certain level in the classification of industries. Below this level, further sub-divisions of industries (e.g., by products) can be made but, in view of the prevalence of joint production, the corresponding net outputs cannot be calculated precisely. For example, one category in the industrial classification is "Textiles, footwear and apparel". This can be divided into a number of separate industries according to the classification of the census of production. The industries might be: cotton spinning, cotton weaving, wool and worsted, footwear, clothing, and so on. A weight representing value of net output can be assigned to each of these industries.

Sub-divisions of the industries, broadly by the nature of the product, can be made for many purposes, i.e. clothing can be divided into men's outerwear, men's underwear and so on. But accurate values of net output cannot be assigned to such subdivisions since the cost of materials, fuel and other services cannot always be allotted completely between them. The first stage, therefore, consists of selecting industry groups, and of allotting the appropriate weight (value of net output) to each, according to the classification of the census of production. These groups will be called
census industries" (or more simply " industries"). If countries are to carry out the recommendations on scope and classification made in section III, the selection of industries will have to conform to the Standard Industrial Classification of all Economic Activities. This does not mean that countries must adopt the Standard Classification as the basis of the census (though it would be desirable); but it does mean that the classification of the census must not conflict with the Standard Classification.

41.   The list of industries selected should be as detailed as possible i.e., the total field of net output should be split into as many small cells as the census data permits. This is to reduce the areas for which individual series have to be selected to represent work done. The field can be divided both by type of end product and by stage of manufacture. Thus the group "Textiles, footwear and apparel" can be divided according to products of cotton, wool, jute and so on, and also according to stages such as spinning, weaving and finishing. For some industries the census data will not lend itself to a split by stage, particularly where there is a long period of production not naturally divisible, as in ship-building and construction. In these cases alternative steps have to be taken to achieve the same end (see section VIII below).

42.   In accordance with the analysis of section II, the weight for each industry is its net output in the sense of its contribution to gross national product in the base period. The definition of net output is a matter of some importance. The figure usually extracted in the analysis of a census is not net output in the sense required for the weights, but something rather broader which can be called "census value added". In most cases, "census value added" is the selling value of gross output less the cost of materials and fuel used, though certain other deductions may also be made. Hence, "census value added" needs adjustment to give value of net output.

43.   First, if it includes any costs of business services such as advertising and insurance, these should be deducted, since such services represent contributions to national product of factors outside industrial production and in so far as they are not used proportionally by all industries, their inclusion distorts the relative importance of different industries. It may be noted that their inclusion in "census value added" does not affect the "netness" of that measure within the field covered by the census; but it is not net in respect of the whole economy. If, therefore, the index of industrial production is added to similar index numbers for other sectors such as distributive services, then weights obtained from "census value added" overlap with weights included in another index and services supplied to industry would be included twice in the total. The compilation of an index of total production, though not considered here, is desirable in itself and provision should always be made for it.

44.   Second, "census value added" should be adjusted where necessary to include the value of any change in work in progress over the census period. This adjustment should not include changes in stocks of raw materials on which no work has been carried out in the industry. Moreover, it should be a valuation of changes in work in progress and not a change in value of work in progress. Many censuses of production do not collect the necessary information, except for some industries (e.g. ship-building) where it is very important; their scope would need to be extended or supplementary data would have to be obtained for the purpose.

45.   In addition, "census value added" may need adjustment to a factor cost basis (excluding indirect taxes less subsidies) and depreciation allowance should not be excluded. When adjusted in all these respects, "census value added" gives what is needed for weighting: i.e., value of net output equal to the selling value of gross output minus the cost of materials, fuel and all purchased services, plus the value of changes in work in progress, all valued at factor cost and inclusive of depreciation allowances. Further notes on the relationship between "census value added " and "contribution to gross national product" are given in Appendix C.

46.   A relevant point is that only relative values of net output are needed. The weights are proportional to values of net output; otherwise the actual values of net output do not matter. Hence, though "census value added" may differ from the value of net output in the sense of contribution to national product, this would not matter if the two sets of values were in the same proportions. Moreover, though the proportions may differ somewhat, this may not affect greatly their use as alternative weights. It is a well-known property of the weights in an average that a moderate variation in weights has little affect on the average, unless the variations are correlated with variations in the direction of movement of the series averaged. Hence, it may sometimes be possible, as a first approximation, to use a census value added" to represent net output on the national product basis in the weights of the index of industrial production.

47.   It cannot be expected, however, that different industries will use outside business services to the same relative extent or that changes in the value of work in progress will be equally important. Efforts should be made, therefore, to obtain correct estimated values of net output as weights, adjusting "census value added" in the necessary respects, by extending the data collected at the census of production or by making supplementary inquiries. It is recognized that this may not be practicable in some countries at present. It is suggested that in these countries values of net output (contributions to national product) should be estimated as closely as possible for the weights of main categories in the industrial grouping, e.g. for such categories as "mining and quarrying" and "food, drink and tobacco". Within each category, relative weights for census industries can then be fixed in proportion to "census value added". The range of approximation is confined, in this way, to relative weights within groups while the correct weighting is adopted as between the groups themselves. Only if this is not possible should use be made of the more approximate method of using relative "census values added" throughout to represent relative net outputs.

48.   In some countries at some periods neither a complete census nor an adequate sample survey of production is available; elsewhere, although censuses are taken, data

are not collected on value added. If an index is to be constructed, the object must still be to use weights reflecting relative net outputs. It must be admitted that in the absence of census data the most careful estimates of weights are hazardous but it may be possible to gather enough information for the compilation of an interim index of approximate accuracy. The values of gross output may be known for the industries used in the first stage of the weighting process. Various devices may then be used to estimate values of net output. For example, there may be available some partial information on costs of materials, or a knowledge of the proportion of net to gross output in some industries, or sample inquiries may be made on these questions; or (at least) data maybe used on proportions of net to gross output in industries in other countries known to be similarly organized. Other possibilities are to use wage bill statistics (if available), or data on numbers employed in different industries multiplied by estimates of net output per head derived from the corresponding figures for similar countries. All this may be very rough and ready; it can only be accepted in default of census data. Estimates of weights so obtained are not likely to be good substitutes for data derived from censuses of production taken with full coverage and with adequate detail on outputs and costs.

49.   It is concluded that, as a first stage in the development of weights and series, convenient groups of industries (census industries) should be chosen in as much detail as possible from the classification of censuses of production. Each census industry is to be weighted with value of net output in the sense of contribution to gross national product at factor cost. If a census of production is available, but the data collected is inadequate for valuation of net output in this sense, estimates of the correct weights should be attempted for the main categories of industries while relative weights of census industries within each main category can be approximated from "census value added" data. Only if census data are not available should resort be made to cruder methods of estimating net output, and the resulting index is likely to be only approximate.

VI. WEIGHTS AND SERIES: TYPES OF SERIES

50.   At the second stage in the definition of series and the assignment of weights, the problem is to obtain quantum series to represent work done in each census industry, i.e., to "extend the net output weight assigned to the industry. Such a series would be expected ideally to measure changes in net output at constant price margins per unit of work done. It should take account of:
     (a) variations in types and qualities of products made, and of materials used, in the industry,
     (b) changes in work in progress, including stocks of intermediate goods
     (c) changes in the amount of processing applied to materials (i.e.,in technical input-output relations.
Materials should always be understood to include fuels, electric power, packaging and business services supplied by other industries.

51.   In fact net output needs to be split into the product of price (p) and volume (q)so that by holding p constant, we can isolate changes in q over time. These factors can be distinguished conceptually but they are not amenable to statistical definition or measurement as separate entities. If the value of net output is an aggregate over many

products, there is always the familiar problem of index numbers-a change in aggregate value cannot be split uniquely into a price change and a volume change. Here, however, it is important to realize that the problem arises for reasons quite apart from the need to split a value aggregated over many products. It arises even in the case of a single establishment in which a single raw material is converted into a single product under conditions of changing technical relationship between input and output. An electric power station employing coal or fuel oil provides an example which approximates to this simple case. It might be suggested that an index of change in the net volume of output of such an establishment could be devised by subtracting input at constant input price from output at constant output price. But such an index can be constructed with base year prices or with given year prices and the two forms will differ except when material and product prices happen to change in proportion. The difficulty arises because of the need to subtract two different valuations at constant prices and this cannot be done uniquely (see appendix D, paragraph 2). In the general case the concept of work done involves a process of transforming several raw materials into one or more final products. The proportions in which the materials are combined or the products are produced will usually vary in response to changes in relative prices, in technological conditions and in other factors. This alone introduces an element of indeterminacy into the measurement of changes in the volume of net output at constant prices. The fact that the prices of products and raw materials are held constant does not by any means imply that the margin between them is thereby also held constant because of the dynamic changes which are continually taking place. Therefore, any statistical formula that could be suggested only represents an approach to our concept of changes in the volume of work done and not a direct measure of it.

52.   Let us assume now that we have complete data for a given industry with respect to products and materials and their prices such as might be obtained from a full census of production. The measurement of work done in the industry
in one period as compared with another can be approximated statistically by an index derived from valuations in each period of net output at constant prices of products and materials used as proposed by Geary (see item 9 of appendix A). This index is constructed from separate gross valuations of output and of materials used. For the given

industry we need a measure of $q_{01} = \dfrac{q_1}{q_0}$ which appears in the form form Q_{01} of paragraph

29. The measure proposed by Geary is

$$q_{01} = \frac{\Sigma P_0 Q_1 - \Sigma \pi_0 \mu_1}{\Sigma P_0 Q_0 - \Sigma \pi_0 \mu_0}$$

 when base year weights are employed, and

$$q'_{01} = \frac{\Sigma P_1 Q_1 - \Sigma \pi_1 \mu_1}{\Sigma P_1 Q_0 - \Sigma \pi_1 \mu_0}$$

when given year weights are employed, where P and Q denote price and output of a product, π and Φ price and consumption of a material, suffixes o and 1 the periods compared and Σ a summation over all products or all materials in the industry. The fact that there are these alternative forms exemplifies the element of indeterminacy involved

in the attempt to measure changes in the volume of work done. In extreme cases, if considerable substitution between materials takes place in response to large relative price changes, the numerator of q_{01} (or the denominator of q'_{01} can actually yield a negative value, which would imply that a negative amount of work was done in one of the periods. An interesting example of an extreme case of this kind has been cited by Dobb (see item 5 of appendix A).

53.   In spite of this element of indeterminacy, which can only become serious in extreme cases, the Geary formulae (or preferably some "cross" form of them) provide statistical measures which generally approach most nearly to the concept of changes in the volume of work done. This can be seen when they are assessed in relation to the considerations of paragraph 50. Variations in types and qualities of products and materials are allowed for because each variety can be included separately in the computation of the index. Next the index can be constructed so as to allow for changes in work in progress by means of the inclusion of intermediate goods. Finally, the index allows, at least in part, for changes in the amount of processing applied to materials. One important advantage of the Geary formula is that it can be computed as it stands, not only for a single industry, but also for a group of industries or for all industries taken together. In the last case the form q_{01} becomes the final index required, Q_{01} without modification. These points are discussed further in appendix D.

54.   The practical difficulties in the way of a computation of q_{01} however are great. Generally, the outputs Q will be confined to final products (indeed to principal products) and the consumption data will be limited to the main materials. In practice, therefore q_{01} is not likely to take account of changing work in progress or of the use of business services. Even so, the data required for compiling q_{01} are unlikely to be available except from a census of production or an extensive sample enquiry. In practice, the series may be available annually, if censuses or sample enquiries are taken as frequently as this, and after some time lag. It might be approximated crudely on a quarterly basis. But it cannot be expected to be available either promptly or as frequently as monthly. It is best considered in relation to annual index numbers, and to annual adjustment of monthly index numbers, in Section IX below.

55.   The main index considered here is one which is to be available promptly and frequently so that attention must be concentrated on data available for periods between censuses of production. Since such data are generally inadequate for the computation of an index in accordance with the Geary formula, any series selected to represent changes in work done in an industry should be regarded as a substitute series. This is the main reason why net output needs to be sub-divided as far as possible at the first stage. The closeness of approximation of substitute series is likely to be improved if the sector covered is smaller (e.g. cotton weaving) than if it is larger (e.g. textiles). In practice two types of substitute series can be constructed from the less complete data available for inter-censal periods:

(a) Amount or quantum of product completed at a selected point in the production process, taken as an approximation to work done in the prior production stage (output series).

(b) Input of labour, materials, energy, etc., during a selected stage of production, taken as an approximation to work done in the stage (input series).

56.   The use of output or input series to represent work done raises many difficulties. The nature of them is seen by simply quoting some of the series which may be suggested for an ordinary industry such as clothing. Work done in the clothing industry can be approximated:
      (a) by output of finished articles of clothing in physical terms (e.g. by number of units or by weight) or in the form of value of output of clothing deflated with an index of clothing prices.
      (b) by labour input measured as numbers employed (averaged over a period) or
      (c) as number of man-hours worked; by amounts of cotton, wool and other cloth used (e.g. in square yards) and of leather used (e.g. in tons), or by the consumption of energy in the clothing industry.

Each of these series will give different answers and none will exactly measure work done.

The difficulties are even more acute in some special cases such as construction, shipbuilding or engineering. In residential construction, for example, an output series generally relates only to the number of completed houses of various types and this may have little relation to work done. An input series of materials used may be no nearer to work done since the materials may be "locked up" in unfinished houses or in pre-fabricated components. For the same reason, a constant price valuation of output (completed houses) less materials used can be misleading; indeed, such a valuation may give a negative figure if there has been an increase in work in progress.

57.   The problems in the main arise because of the three factors mentioned in paragraph 50. Each of these factors affects in different ways, the approximation which different series bear to work done. Their effect may be illustrated by setting out the difficulties arising from the employment of a series in common use, namely an output series measured in some simple physical unit. Firstly, the quality or type of product may change without showing up in the physical units (e.g. finer clothes in a series in square yards). The solution here turns on carrying different series or constituents for varying qualities and types, i.e. attention should be directed to the definition of the product. Secondly, there may be changes in the amount of work in progress during a reporting period (i.e. the work in progress at the end of the period may differ from that at the beginning) which would not show up in the output series. This will not cause difficulty if the change in one period is the same as that in another. It is not the existence of stock-piling which causes the trouble (since if the rate of stock-piling is a constant proportion of the rate of output, the output series adequately represents the stock-piling) but changes in the rate of stock-piling relative to output. The difficulty is partly overcome, but not completely, by taking output data at various points in the production process. This makes possible the inclusion of stock-piling at the selected points but ignores changes in intermediate work in progress. If significant changes in work in progress are to be expected, as in construction, shipbuilding and engineering, then a series other than an output or input series should be sought. Thirdly, the amount of processing applied to materials per unit of product may change quite apart from variations in the quality or type of product. Materials of a greater or less degree of fabrication can be used or outside

"business " services can be used to a greater or less extent. In the clothing industry, for example, there is a choice between rough or more finished leather as a material. Again, in residential building, many house components can be constructed by the industry on the site or they can be bought as pre-fabricated units from other industries. There is no real solution for this difficulty in the short-run, i.e. between censuses. All that can be done is to keep to a minimum the period during which the unadjusted output series is taken to represent work done.

58.   Some of the difficulties are more important for single establishments than for an aggregation of establishments in an industry, and more important for a single industry than for an aggregation of industries into a total. Variations in the amount of processing applied to materials, for example, may cancel out an aggregation, one establishment or industry increasing the amount of processing at the expense of another in the next stage of production. However, the difficulties never completely disappear, even in the aggregate of all industries, as long as there are imports of materials which can vary in degree of fabrication.

59.   Output series can be obtained directly in physical units or by deflating value of output with the aid of a suitable price index. Both forms present problems which arise from a basic difficulty, namely that the output of an establishment or an industry is very seldom homogeneous. It embraces products of different types (as different cotton, rayon or woollen dresses) and products which vary in quality (as different qualities of chambray or broadcloth).

60.   If output is measured in physical terms, there are various alternative units which can be used. A choice can be made quite often between a count, a volume, area or length measure, and a weight. There may be other measures, such as horsepower and engine capacity for machinery or vehicles. A series of output of cotton cloth by square yards will generally differ from a series by weight, because of varying types and quality of product. A series of output of automobiles by number will generally differ from a series by weight, by passenger capacity or by horsepower. Neither the measure by square yard nor that by number is the quantum series required because they do not take account of quality changes. It may be that the output in some cases is so nearly homogeneous and free from quality changes that any one physical unit will give the required quantum series. In a few cases, moreover, a special unit of measurement can be used to eliminate most of the relevant quality changes. This may be true, for example, of ores aggregated by metal content or alcoholic drinks by alcohol content. These are, however, special cases and their range is narrower than is sometimes supposed. In general, the product is so variable in type and quality that no one physical unit can be found to serve as a quantum series.

61.   The solution to this difficulty is to separate the different types and qualities and use separate series or (what amounts to the same thing) to devise some quantity index to cover the varying qualities. The alternative is to take the value of output of various types and qualities and to deflate with an index representing changes in the level of prices of output, for example, a series of value of output of clothing can be deflated with an index of clothing prices. These two methods are not basically different, but rather alternative ways out of the same difficulty. The problem of measuring changes in an industry with

non-homogeneous products, is the same as the problem of measuring changes in the whole of industrial production, some solutions to which are described in paragraphs 29 and 30. The different forms, base-weighted and current-weighted, of the volume index match off with the opposite forms of the price index to yield the value index as a product. Therefore, either a base-weighted volume index can be computed direct, or a value index can be computed and divided by a current-weighted price index, to produce the same answer. There is no choice between the two methods except on the basis of practical convenience and probable accuracy. Certainly, unless the product is completely homogeneous, a physical series is not inevitably superior to a deflated value series. Indeed, since in practice both calculations are likely to be based not on a complete coverage of the product but on a selection of types and qualities, the deflated value series (if available) may be preferred, because it is usually more reasonable to assume that the prices of items omitted from the index move in the same way as the prices of the items included, than to assume that outputs move together. The practical difficulty, however, is to obtain the requisite data. First it is difficult to obtain price quotations completely appropriate to a value series. Second, it is difficult to obtain the "quantities needed for weighting the price index, since they should be current quantities. In practice it is likely that only base period quantities will be available, and that only a base-weighted price index can be computed for deflating. It will be justifiable to use such an index only for interpolating between two benchmarks between which the Laspeyres and Paasche forms of the production index do not diverge significantly.

62.   The output series used, whether in physical or deflated value terms should represent <u>production</u> of completed items at the end of a stage of production, e.g., production of cotton cloth or automobiles. The series should not be <u>deliveries</u> to the next stage of production (as with cotton cloth) or to consumers (e.g. automobiles). The figures needed represent the result of current production, whether for sale or for stock. Deliveries, however, are made both from current production and from stock and they represent the result partly of current and partly of past production. If the timing of production figures is right, then the timing of deliveries is wrong.

63.   <u>Input series</u> are less homogeneous than output series, and the different types are in differing relationship to work done. The main series in use refer to the input of labour, materials or energy. In many countries, the most generally available is a labour series such as numbers employed or man-hours worked. Between these two preference is to be given to a series of man-hours worked, since it takes account of short-time and overtime working. Even if man-hours are used, however, there may be need for some adjustment to allow for changes in the proportions of men, women and juveniles employed. It is sometimes possible to make such an allowance, or to overcome the difficulty of lack of man-hour data, by taking a series representing the aggregate wage bill in an industry and deflating with an index of wage rates. This is not of general application, however, since it introduces a new complication. The wage bill generally includes overtime payments calculated at more than straight-time rates. Consequently, changes in overtime work as a proportion of total man-hours worked would create distortions in the derived index of man-hours worked.

64.   The advantage of labour input series is that they are fairly direct approximations to series of work done. In general, for example, timing of labour input and of work done

agrees though the proportions of work done in different stages of a productive process may not be the same as the proportions of labour input. The main difficulty, and the one which prevents a general use of labour input series, is that they do not take account of changes in the productivity of labour in the sense of output (or, rather, work done) per man-hour. Such series can only be used as an approximation to a series of work done if it is known that changes in labour productivity in an industry are small. If labour input series are employed at all extensively in the construction of an index, the index cannot be used for the purpose of assessing changes in the productivity of labour. This is very serious since one of the uses of an index of production is to throw light on this important question. It follows that, as a general rule, limited use of labour input series may be justifiable in the short-run, e.g. for a monthly index over a period of a year or so. For use over a longer period they would need to be adjusted for changes in labour productivity.

65.   To use series of input of materials involves the assumption that net output is constant per unit of materials used. This is not plausible where many different materials, together with fuels, packaging and business services have to be taken into account. It can be accepted only for an industry where one homogeneous material (or, at most, a few materials) accounts for the bulk of materials used. The series should represent the amount of the material consumed (not purchased), measured in physical units. Good examples are consumption of flax (in pounds) in the linen industry and consumption of newsprint (in tons) in the production of newspapers. Where there are several materials, some allowance or adjustment needs to be made for changes in the proportions used in production. One possibility would be to take a series of values of all materials used in the industry, deflated with an index of the materials prices.

66.   The disadvantage of materials input series, is that, unlike labour input series, they are far from a direct representation of work done in an industry. Indeed, materials input series have most of the limitations of output series and are seldom to be preferred to them. The timing of materials input, even if measured strictly as consumption of materials in an industry, is not that of work done. Such a series may allow to some extent, but by no means completely, for changing types and qualities of products. A series of materials input does not make a correct allowance for changes in work in progress; in fact, while an output series errs in one direction, an input series tends to err in the other direction. For example, if there is a growth in work in progress in a recording period (e.g. stock-piling of intermediate products) and if the growth increases from one period to another, then some part of the materials used is being "locked-up" in partly finished product. In such a case, the materials input series rises more, while an output series rises less, than work done. Materials input is also an imperfect representation of work done when there are changes in the amount of processing applied to materials for a product of given type and quality. For example, if cruder materials or less fabricated components are purchased by an industry and more work done on the materials and components in the industry itself, then more work is done and less materials are used for a given output. Hence, when work done is increasing, it may be found that an output series remains unchanged and a series of materials input actually declines. There are additional, though perhaps less important, difficulties with a materials input series. It will ignore technical substitutions of minor for major materials if it is confined to a few of the more important materials. Changes in the amount of wastage of materials may not be adequately allowed for in a series of materials recorded as used or consumed.

67. A series based on consumption of energy would appear to have some advantages. As with man-hour data, energy series of a single type can be constructed for diverse industry groups and there is a convenient and standard unit of measurement. The timing of the series would be better than materials series though probably not as good as labour series. The energy series used must be total consumption of energy, whether purchased or produced on the spot. There is a difficulty here, since the available data are often confined to purchases. The main difficulty, however, is that the relation between consumption of energy and work done or output is peculiarly liable to change. The introduction of new machinery, for example, will often have a much greater effect on energy consumed than on labour and material inputs or on output of product. If no other series is available energy series can be useful in interpolating between bench-marks based on further data. Special care must be taken however to observe and allow for technological changes affecting energy consumption.

68. It is concluded that the complete data necessary for an accurate index of changes in the volume of work done are unlikely to be available even from a census of production. An approximate series, namely valuation of net output at constant prices of products and materials, can be constructed from the data available in many censuses of production, and this series is much nearer to work done if the product component includes goods in process of manufacture and the materials component includes business services. Between censuses of production the possibilities are generally limited to cruder measures, namely output or input series.

69. Output series should represent production, not deliveries. If the product is reasonably homogeneous, a direct physical measure can be used and the units selected for measurement make little difference. If the product is not homogeneous, an output series, allowing at least in part for the changing composition of types and qualities, can be obtained either as a base-weighted volume index or as a deflated value series using a current weighed price index if one can be devised. Such a series makes little or no allowance for changing work in progress or changing amount of processing applied to materials. The first difficulty may be partly met by splitting up the production process, but the second cannot normally be met from the data available between censuses. Output series therefore represent work done only approximately and are valid only as long as changes in work in progress and in the amount of processing applied to materials are known to be small.

70. A series of materials input as a representation of work done is subject to much the same limitations as an output series and the approximation may well be less close. A materials series should be considered only if the variation of product by type and quality makes it difficult to derive an adequate output series and if a single homogeneous material is the main material used in production. Even under these circumstances, it is necessary to check, as with output series, that changes in work in progress between the input and output stages and changes in the amount of processing applied to materials are not large. A labour input series, particularly when measured in man-hours and adjusted for major changes in proportions of men, women and juveniles, is not subject to these limitations. The major difficulty with this series, however, is that changes in productivity of labour are not allowed for. Labour input series should be avoided in large

sectors of industry when productivity changes are to be expected and, when used, should be confined to short period movements, say monthly over a period of a year or two. Finally, if no other series are available, series on the consumption of energy can be useful over short periods, in industries where it is known that no important technological changes affecting energy consumption have taken place, or where such changes can be allowed for.

VII. **WEIGHTS AND SERIES: CHOICE OF SERIES AND OF WEIGHTS WITHIN INDUSTRIES**

71.   Two general observations can be made on the choice of weights within industries, and on the choice of series to represent work done. First, accurate estimates of net output are not generally available within a census industry. However, as long as net output weights are fixed as between census industries and therefore the main divisions of the weighting scale are correct, the use of rougher weights within a census industry may not affect the index greatly in total or in its broad constituents. It follows that, in fixing internal weights within an industry, various approximations can be used; weighting can be by rough estimates of value of net output, by value of gross output, by employment or man-hours worked, and by similar measures of relative importance. Little attention seems to have been given to this problem and it is not possible with the available data to assess accurately the relative merits of these alternative criteria of relative importance. The measure most commonly used at present is apparently relative values of gross output but it is by no means clear that it is the best for all or even most census industries. Frequently, differences in gross value per unit of output reflect mainly differences in costs of raw materials per unit rather than differences in work done per unit. In such cases the use of man-hours worked is preferable if the data are available. Further research and experimentation are clearly required in this field. The available evidence suggests that no single measure of relative importance is likely to be generally applicable but rather that each industry will have to be treated as a separate problem.

72.   Secondly, on choice of series, a considerable degree of flexibility is both possible and desirable. There may be alternative series available for an industry or a sector of an industry. For example, in brass production, possible series may include output of brass products, consumption of copper and man-hours worked in brass production. The choice between such alternatives has to be decided on the merits of each case; they are all substitute series approximating more or less closely to work done. There is no rule-of -thumb method for selecting the series which approximates most closely. The choice must be made in each case from knowledge of the industry and of the data. As between various sectors of an industry, choice must be left free for the use of one type of series in one sector and another type in a second sector.

73.   It maybe practicable, for some census industries, to select a single series to represent adequately the movement of work done in the whole industry. The series may be output in physical terms, e.g. pig iron for the blast-furnace section of the iron and steel trades. It may be an input series covering the entire industry, e.g. consumption of flax for the linen industry or man-hours worked for the glove industry. The decision here is simply

whether to choose an input or an output series. More commonly, however, the census industry will be so diversified in its products and in the materials used that no single series will suffice to represent work done. Two or more series have to be selected and combined into an indicator for the whole industry. It may be that several series of the same type (e.g. of output or materials input) can be used for the industry. It is not practic-able, nor even desirable, to include all the diversified outputs (or materials); the problem is to make a selection of series and to choose a set of weights for combining them. This is a straight-forward problem of index number construction which may be solved directly by compiling a volume index covering various types and qualities of product (or materials used) or indirectly by deflation of a value series. For example, in the lace industry, outputs of selected lace products may be combined into a volume index, or total output of lace by value may be deflated with an index of lace prices. In either case, a selection of lace products or prices has to be made and weights fixed for the volume or price index. Alternatively two or three materials can be selected and combined into a series for the industry. The decision here is not only whether to select input or output series but which items to include and how to weight them.

74.   More usually, a single indicator cannot be selected or devised for the whole industry, which must be divided into a number of sectors and approximate weights settled for each sector. This may be because the sectors are very different in the nature of the product, or because the kind of information available varies from one sector to another. For example, in the group of "basic metal industries", one census industry may be smelting, refining and basic manufacture of non-ferrous metals. This is best divided into sectors (copper, brass, aluminium and so on) and output data may be available in some sectors (e.g., output of refined copper) while only input data are available in other sectors (e.g., consumption of zinc concentrates or employment in brass production). In some sectors both input and output data may be available and a choice has to be made between them. Occasionally it may be decided to represent a sector by an index constructed from two or more series, but unless it is a very important sector this will not usually be worth while.

75.   In the examples considered so far, data have been available for covering all sectors of the industry, either by a single series for the whole industry or by separate series for each sector. Frequently, however, data are available for some sectors but are almost completely lacking for others. For example, in the basic non-ferrous metals industry, output or input data may be available for copper, aluminium, lead, etc., but no suitable series can be devised for brass production. Again, in the footwear industry, it may be that a fully adequate series can be found only for leather boots and shoes while very rough or scattered data exist for slippers and for rubber footwear and nothing at all for footwear partly or wholly of fabric. It is in cases like this that most problems arise. The gap can only be filled by some form of imputation.

76.   There is a good deal of choice between different methods of imputation, for example between assigning the weight attached to one sector to that of another and «blowing up» a series available only for one sector to represent the whole industry. Whatever method is adopted, there must be an explicit assumption that two or more series behave similarly with respect to some character. The assumption can take many forms but three of them are of general applicability. It maybe assumed that variations in

(a) output

or (b) price of product

or (c) productivity of labour (output or work done per man or man-hour)

in the sector in which data are lacking follow the variations in the sector which is covered. No hard and fast rules can be laid down on the choice to be made in particular cases between these assumptions. Assumption (a) is the most familiar. However, variations in price are as likely to follow each other as variations in output. Further, changes generally occur more slowly in productivity of labour than in output so that it may often be more reasonable to assume that two sectors have similar productivity variations (i.e. both small) than that the output changes are similar. Imputing the weight of one sector to another, and using the output series of the second to represent changes in both, may not therefore be the most reasonable method to adopt.

77. The method of imputing the weights of an uncovered sector differs according to which character (output, price or productivity of labour) is taken for the assumption of parallel movements. First, it may be assumed that the parallel movements are in output. It then has to be decided whether output in the uncovered sector is likely to vary with output in one of the covered sectors of the same industry, with the combined output of two or more covered sectors (either within or outside the same industry), with output in the whole of the rest of the industry, with output in the rest of a group of industries, or with output in the rest of the whole field of industrial production. According to the assumption made, the weight of the uncovered sector is added to the weights of the sector or sectors, industry or industries which are assumed to have parallel movements. When the weight is imputed to the total weight of the index, the uncovered sector can be omitted altogether for computation though it is conceptually included. These are the more common cases of imputation but there are others, all following the same general plan. As an example, there may be no suitable series available for one sector, brass production, of the basic non-ferrous metals industry in the group "basic metals industries". The weight for brass production may be imputed to the copper sector, or spread over copper and zinc in the proportions in which these metals are used in brass production. Or, the brass weight may be imputed to some sector such as miscellaneous metal products outside the group of "basic metals industries". It may however be preferable to impute the weight for brass production to the rest of the basic non-ferrous metals industry, or to the rest of the group of "basic metal industries", or to the whole of industrial production. The choice depends on what can be most plausibly assumed about the variation in brass production.

78. When the weight of one sector A is imputed to that of another sector or sectors B, there are alternative ways of showing the imputation in setting up the details of the index of industrial production. The description of the sector or sectors B can carry a note that the weight includes the weight of sector A which is not otherwise included. Alternatively, both sectors A and B can be shown in the index and the series for A is given as a repetition of that included under B. Suppose, for example, that the weight for brass production is distributed over copper and zinc production. Then the weights for copper and zinc can be shown as each including a part of the brass weight, with brass production not shown separately. Or brass production can be set up as a separate entry with a series which is a weighted average of the series of copper and zinc outputs. The second method has much to recommend it since it makes the nature of the imputation

clear and since it avoids transferring a weight across the boundary of a census industry or group.

79.   The next assumption is that changes in price of product are similar. As a simple case, which can be easily extended, suppose that one sector A in a census industry X is not covered by series representing work done or by any price and quantity data while the rest of the industry (Sector B) is covered. A price index can be computed for sector B and applied equally to sector A and to the whole industry X. A quantum series for the industry X may then be obtained by deflating the value of total output in X with the partial price index. Or, if the value of output in sector A is known separately, this can be deflated with the partial price index (from sector B) to give a quantum series for sector A. Such a series can be used in conjunction with other, and quite different, series for the covered sector B. For example, the value of total output of non-ferrous metals can be deflated with a price index computed from all sectors except brass production. Or, if the value of brass production is known, without details on price and quantity, then this value series can be deflated with a price index drawn from other non-ferrous metal prices and used as the quantum series for brass production.

80.   Finally, the assumption adopted may be that productivity changes are similar. With the same simple case as in paragraph 79, suppose that employment or man-hour data are available for the census industry X and separately for each of the two sectors A and B but that the labour input series, which could be devised, is not acceptable for the industry since changes in productivity of labour are known to be taking place. Suppose further that output series are appropriate for the industry but that the only series available are for sector B. The output series for sector B can be used to derive an index of productivity of labour for that sector. The latter can then be used to adjust the labour input series for sector A. Employment can be taken either as average number employed or as man-hours worked. In the former case, the assumption is that productivity in the sense of output per man varies in the same way in sectors A and B. In the latter case there is a similar assumption in terms of productivity as output per man-hour. For example, series for copper, aluminium, etc. may be combined to cover the whole of the basic non-ferrous metals industry except brass production. The combined series can be used to derive an index of labour productivity which in turn is used to adjust the employment series for brass production. Hence, even if a labour input series is not considered appropriate as a quantum series in itself, it is still possible to make valid use of employment data to fill gaps in output series available for an industry or group of industries.

81.   One problem, which is likely to arise frequently in practice, does not correspond entirely to any of the examples taken above. This is the problem presented when a group (or an industry), for which a somewhat unsatisfactory input series is readily available, is divided into several industries (or sectors) finely enough for output series to be adequate representations of work done but for only some of which adequate output series are available. The problem is to decide between the two alternative approaches: either to take the limited but good output data and to "blow up" to complete coverage, or to use the inferior but comprehensive and readily available series of inputs. An example is the group of machinery industries for which good output data are generally limited to a few particular types of machines. It may be possible to use employment or man-hour data, or materials input series based primarily on steel consumption, but close approximations to

work done may not be obtained. Hence, it may be better to concentrate attention on the more difficult method of filling out the gaps or of "blowing up" the available output series.

82.   Many countries will have to use all these types of imputation fairly extensively because they lack current data or because they wish to avoid using too many current series. However the device must always be used with restraint. For instance if the available data for the foodstuffs industry are limited to the output of flour, sugar and margarine it is more than doubtful whether the combined index for these series should be given the weight of the whole foodstuffs industry. The assumption of parallel movements, particularly of output but also of prices and productivity, should be made only if there is real reason to expect a parallel movement, not merely absence of evidence to the contrary. However cleverly contrived the system of weights, a given amount of current data can only be stretched just so far. If this is not far enough to provide a comprehensive index, no general index should be given and as is concluded in paragraph 26 the groups for which sub-indexes are given should be confined to those properly covered.

83.   Particularly difficult problems in the choice of series arise in an industry with a long period of production not easily divisible into stages. The main examples are construction and shipbuilding, but others are production of aircraft, locomotives, electric generating plant and elaborate machinery of all kinds. In such an industry it is not generally acceptable to take either a series of completions or deliveries e.g. of residential houses of merchant ships, or an input series e.g. of materials or labour to represent work done. The general solution is to break up the industry into stages and to devise quantum series for each stage. This cannot be done directly since there are no definite stages e.g. in house construction or shipbuilding where the intermediate product is passed on from one sector of the industry to another. Some less direct way of sub-dividing the industry must be sought

84.   One method is to define certain arbitrary stages in the production process, which is continuous, and to estimate the average proportion of the total work which is performed during each stage. For example, in residential house-building, the stages may be (a) from breaking ground to "damp course" level with (say) 10 per cent of the work in the stage, (b) from "damp course" level to "eaves" level with (say) 25 per cent of the work and so on to completion. Regular monthly, or at least quarterly, statistics are then needed on the number of units within each stage, if possible distributed according to the length of time spent within the stage. From this information can be computed the equivalent number of completed units on a work-done basis. A variant of this method starts from an estimate of the length of the period of construction, made for each type of unit at the time work commences. The total period is divided into stages of one month and the average proportion of the total work achieved in each month is estimated. For example, in shipbuilding, the length of the period of construction for a certain type of merchant vessel may be 10 months and it may be estimated that 5 per cent of the total work is done in the first month, 10 percent in the second month, and so on. Regular figures are then needed on the number of units of each type under construction, distributed by the length of time elapsed since commencement of work. Again, the equivalent number of completed units can be computed month by month.

85. Special computations of work done on this basis are sometimes possible, and should always be aimed at, for the major sections of construction and shipbuilding. The results can be combined with other and rougher series calculated for the minor sections of the two industries. Series of materials or labour input or deflated value series may suffice, for example, for construction of non-industrial and non-residential buildings, for building repair, conversion and adaptation and for ship repair work. Special computations of work done may also be possible for other long period industries such as aircraft, locomotives and machinery. In general, however, they are both less practicable and less essential in these industries. It is suggested that special computations approximating to work done should be made at least for series covering the more important sections of construction and shipbuilding.

86. It is concluded that, within a census industry, there should be flexibility in the choice of series representing work done in different sectors and that the internal weights for combining these series can be more roughly estimated than in the main weighting system. The method of filling gaps in coverage of series by the use of imputation should be used only where there is real reason to expect parallel movements, and particular attention should be given to the question whether the parallel movement is assumed to be in output, in price of product or in productivity of labour in various sectors. Specially computed approximations to work done should be derived as series for the more important sections of the construction and shipbuilding industries.

VIII. COMPILATION

87. Once the weights are fixed, the base-weighted (Laspeyres) index of production is particularly suitable for computation in tabular form. Every month or quartery the figure obtained in each series is expressed as a relative on the weight base as 100, the relatives are multiplied by the fixed weights and aggregated. This work of compilation, month by month or quarter by quarter, depends for its speed and accuracy on the series used. One factor is the number of individual series, some representing industries or sectors of industries with separate weights, others being combined in groups before weighting. For example, a single series of pig iron output may represent iron and steel (blast furnaces) while several series (e.g. outputs of different industrial acids) may be averaged for each sector of the chemicals industry. If there are only a few series, or if some of them have large weights (direct or imputed), then the total and main group index numbers can be unduly influenced by unrepresentative or inaccurate movements in one or two series. Fewer than 100 series may well prove insufficient for an accurate index. On the other hand, the inclusion of a large number of individual series may slow down the compilation to a greater extent than the extra accuracy justifies. It may be preferable to have a less accurate index computed promptly than a more accurate one available only after a considerable interval. A few hundred series are generally to be preferred to several thousand and it should not be necessary in general to include more than 500, though the number depends partly on the number of sub-indexes shown separately.

Whatever the number, no single series should bear a weight (direct or imputed) even,as large as 5 per cent of the total unless it has been carefully examined for suitability and accuracy. Examples of series which may need attention in this respect are output of coal and steel ingots, work done in residential construction and generation of electricity.

88.   A more important factor is the accuracy of individual series. Each series used should be assessed as regards its approximation to a true work done series and each should be graded as to the accuracy with which it measures what it is intended to measure. The grading need be no more precise than some simple qualitative grade (A, B, C, ………) of descending accuracy. For example, one series may be of physical output and a close approximation to work done, but it may be subject to errors in collection and graded D for accuracy. Further, to assess the accuracy of the index in total and in its' main components, each series or group of series should be shown with its weight and with an indication of how much of the weight is direct and how much imputed. When the index is revised, efforts should be made to improve low graded series and particularly those having large weights. For a sector with a small weight, however, an inaccurate series may be rejected in favour of imputing the weight to other sectors.

89.   Another factor is the promptness with which series become available. Speed of assembly will vary for series from different sources and of different types. There is a conflict here between the need for prompt compilation of the total index and the desire to avoid large or repeated revisions in provisional figures. Generally, the compilation should not be held up in any month or quarter because of delay in obtaining a few series, but the index should be shown in provisional form. It is desirable, however, to limit the amount of revision needed. In some cases, therefore, a series which is inferior (in the sense that it does not approximate very closely to work done) may be accepted for inclusion provided that it is available promptly and in readily usable form.

90.   In assessing series according to accuracy and availability, the source of information and the expenses involved in collection are particularly important. Series may be obtained at little expense from data already assembled for other purposes, or they may be derived by the expensive process of collecting special returns from individual establishments. Series of the first kind may not be either suitable or accurate for the index of production. On the other hand, considerations of expenditure of time and money may prohibit the use of special returns, except on incomplete coverage (e.g. returns from large establishments only). The uses of sampling techniques in this field, to maintain accuracy in the index at small cost, are referred to in Section XI below.

91.   It may be possible to improve the accuracy of a series after an interval by adjusting it in the light of information available less frequently than monthly or quarterly. One example arises when the series is obtained monthly or quarterly from returns from part of the field, e.g. from certain establishments only. Supplementary information on the rest of the field can be obtained at intervals, perhaps by sample, and this can be used to adjust the basic series for incomplete coverage. Another example is when the series available monthly or quarterly is only a rough approximation to work done while closer approximations are obtainable at intervals (say annually). If man-hour data are compiled into an input series, then changes (usually slow) in productivity of labour are ignored. Adjustments may be possible in such a series on the basis of annual data on changes in

productivity. When such adjustments are made, there is always the question of the technique to adopt to avoid discontinuities in the series. One adjustment is to apply a fixed percentage factor to each monthly figure in a year, the factor being determined by the relation of the adjusted annual figure to the unadjusted annual average. This would give rise to a discontinuity between each December and the following January figure. Another method of adjustment, which avoids discontinuity in cases in which the monthly and annual series diverge progressively from each other at a more or less constant rate, is to spread the change over the year, starting with a fixed figure for December of the previous year and finishing with an adjusted figure for December of the current year.

92.   A different reason for adjustment is the introduction of new products and the disappearance of old, a process which goes on gradually and continuously. If a completely new industry arises e.g. synthetic rubber or television sets, it is very difficult to adjust an existing index satisfactorily; only a complete revision and reweighting of the index will usually meet the case. In some cases, however, it may be possible to include the new industry in the existing index on a provisional basis pending a complete review. For instance, it could be assumed that the ratio between the prices of synthetic, and natural rubber and the ratio of value added to gross value in the synthetic rubber industry would have been the same in the base period as those in the current period if the synthetic rubber industry had existed in the base period. The ratio of value added to gross value may have to be estimated or ascertained by means of a sample enquiry if a full census has not been taken since the base year. These assumptions could be used to derive an estimate of the net value of synthetic rubber output per pound in base period prices. The index excluding the synthetic rubber industry would be expressed in the national currency by multiplying each index number by the net value of industrial production in the base period. The net value of synthetic rubber production in estimated base period prices would be added to the total for each period and an adjusted index could then be computed. In the more usual case, where new products are arising and replacing old products within an industry, it is generally possible to adjust the index without complete overhaul. The system of weights by census industries can be maintained but, within an industry, the series used and their (internal) weights can be adjusted provided that a link is made at the month of change. For example, the fixed weight of a chemicals industry may be extended by a group of 5 series combined with appropriate weights. In a selected month, one series may be dropped, two new ones added and a revised set of internal weights fixed for the 6 series. This new combined series can be linked to the old series and used to extend the same industry weight as before. Such substitution cannot be adopted too frequently; one of the reasons for periodic revision of the index is to avoid a series of adjustments having a cumulative biased effect.

93.   It is clearly preferable that the index of production computed in the current month (and generally also in recent months) should be calculated on a provisional basis. Some of the series will need to be adjusted in the light of more complete data available at intervals, or to introduce new items. Other kinds of revisions may be proceeding all the time. Examples are series available only quarterly and needing to be spread over months, and compilations of work done in construction or shipbuilding based on provisional estimates of the over-all period of construction. The main reason for a provisional calculation, however, is that some series are not available in time. The

provisional index is then computed with a greater area of imputation, e.g., on the assumption that a missing series moves parallel with some other series which are available. If it is known or suspected that an important omitted series has moved differently from the series included, it is better to make an estimate of the movement of the omitted series than to leave it out of the provisional calculation.

94.   The general index and the group indexes should all be published to one decimal place. This does not mean that even the best indexes of production are regarded as possessing this degree of accuracy. This recommendation is made solely with a view to facilitating the employment of the indexes for purposes of analysis. The publication of all national indexes to one decimal place would make possible more accurate conversions from one base period to another for purposes of international comparisons. It would also be convenient for users who wish to combine several group indexes or to eliminate a particular group from the total index for special purposes.

95.   There are difficulties in compiling and using index numbers of production on a monthly basis, since calendar months are of unequal length and contain varying numbers of week-ends. As a basic time period for measuring production a week, quarter or year are equally convenient and a translation can readily be made (with very minor adjustments) from one basis to another. The calendar month is not a period which fits into this pattern. Further, the weights of the index are generally obtained as values of net output per year (e.g. from a census of production) and these can again be translated into weekly or quarterly but not into monthly rates. It is suggested, therefore, that the primary index of production should be constructed on the basis of production per working week, i.e. that series representing product per week be applied to weights which are relative values of net outputs (equally per week or per year). The index can be shown for "months" that is computed twelve times per year for periods which can be labelled January, February, and so on.

96.   To achieve this some adjustment is needed in the basic data for series. If the data are given for calendar months, which is commonly the case, the adjustment will depend on the normal work-week in each industry. For example, if an industry has a 6-day week with no production on Sundays and other days counting equal, then the number of working days is computed for each month by omitting all Sundays; the figure in the series for the month is divided by the number of days in the month and multiplied by 6 to give the rate per working week. Similarly, for an industry with a 5 1/2 day week (five full days, half Saturday, no Sunday production), each month's figure is divided by the number of working days, omitting Sundays and counting Saturdays as halves, and multiplied by 5 1/2. If the data are given weekly throughout the year, then the output of weeks that fall partly in one month and partly in the next can be allocated between the two months in proportion to the number of working days that fall in each month. The average output per working day is then calculated and multiplied by the number of working days per week to give the average rate per week. Alternatively, each week can be assigned to the month in which it falls, in entirety or in major part; the data are then aggregated for the weeks in the month and divided by 4 or 5 according to the number of weeks. (As a further variant an arbitrary pattern can be set in advance, e.g. 4,4,5, weeks in the three months of each quarter.) Finally if the data are given only quarterly,

some interpolation is needed; this is best done on a weekly basis and the weeks assigned to months as indicated above.

97.   Two points are to be emphasized. The first is that the adjustment of basic data given for series is intended to reduce production to the primary weekly basis, to eliminate the vagaries of the calendar. It is not an expression of production per working day, which will need to allow for public and annual holidays as they occur over the year and for changes in the length of the working week; if, for example the working week were reduced from 6 to 5 days with a proportional reduction in production, the index suggested would show a fall. Nor is it an attempt to isolate the trend of production over time. The other point is that, when the basic data are for calendar months, the adjustment will differ as between countries, and, within one country, as between different industries, because of variations in the normal work-week. It follows that the adjustment is not one to be applied to the total index or to its main components. The choice is between applying it to the individual series or to convenient groups of series. The former is more accurate but also more laborious. The latter can usually be adopted in practice, if most establishments in each industry have the same normal work-week and if industries with the same normal work-week are grouped together

98.   The movements over time in the primary index of production (per working week) are affected by seasonal factors. These include the incidence of public and annual holidays as well as such factors as the effect of weather. Hence, for comparisons involving the trend of production from month to month, it is important to have one or more secondary index numbers from which seasonal influences have been removed. A distinction can be drawn between the incidence of holidays, some of which are fixed while others fall in different months in different years, and the effect of other seasonal factors. The primary index may be first adjusted for holidays, and then a further secondary index obtained by eliminating other seasonal influences. The second adjustment can be made according to one or other of the standard statistical techniques for isolating seasonal variations. The first adjustment, however, needs special treatment. The aim should be to fix estimates of the effective number of working days lost because of holidays each month; in any one country, this may well vary from one region to another and from one industry to another. The decision will be made on general information on what holidays are taken in different industries and regions, supplemented by data on the effect of these holidays on production. For example, the Easter weekend may have the effect of 1 1/2 days of lost production in some industries in England, and 2 or 2 1/2 days in other industries; while in the United States the incidence may be generally equivalent to only 1 day.

99.   The secondary index computed by elimination of the effect of public and annual holidays and allowing for changes in the length of the working week may be described as an index of production per working day. It should be noted, however, that this index is derived from the primary index per working week and
hence that the weights of different industries are still relative values of net output per week or per year. A different index of production per working day, more consistent if the working day basis is to be taken, could be calculated by reweighting with relative importance of different industries and hence the course of the index. For example, an industry with a 6-day week may have 290 working days in the year (allowing for all

holidays) while an industry with a 5-day week may have only 240 working days. The weight of the first industry would be reduced relative to that of the second when switched from a per week (or per year) basis to a per working day basis.

100.   It is concluded that several factors should be considered in assessing the accuracy and availability of the total index of production. The individual series used should be neither too few (say less than 100) nor too many (say over 500) in number. They should be graded according to their individual accuracy and considered in relation to the speed and expense at which they can be assembled. Both because some series are not available in time, and also because series may be adjusted in the light of later data or to introduce new items, the total index for the latest and recent months needs to be calculated on a provisional basis and later revised. The amount of such revision should be limited as far as possible. The general index and the group indexes should all be published to one decimal place. The primary index should be on the basis of production per working week, basic monthly series being suitably adjusted, to eliminate the effect of the varying length of the months and of the varying number of week-ends per month. Secondary index numbers can then be computed with seasonal influences (including the incidence of holidays) removed. However, the primary index shall always be shown and the method of adjusting it to any secondary index should be indicated.

IX.   LONG-RUN COMPARISONS

101.   Attention has been concentrated so far on the compilation of an index of pro-duction at frequent intervals, preferably monthly, for short-run comparisons over time. An index of this type needs to be tested and reviewed from time to time and, whenever it is found necessary, overhauled and reweighted. It has been suggested (Section IV) that the review should be made regularly at five-yearly intervals, if not more often, and in conjunction with censuses or sample surveys of production. Each reweighting of the index then introduces a new series of (monthly or quarterly) index numbers which is not automatically linked to the previous series. Over a long period, therefore, there are several sets of index numbers each suitable for short-run comparisons but no longer-run relations are established. At intervals, perhaps as often as every five years, there is need to link together two separate series of index numbers to form a longer series. This may be done by using the data from successive censuses to calculate an index with a wider coverage, which will measure the change between each two linking points, and adjusting the sets of short period index numbers to conform with the bench-marks thus set up.

102.   These bench-marks may be set up in the following way. A link is required between two years denoted by suffixes o and 1 for both of which full data of the kind obtained in a census of production are assumed available. There is no need for immediate compilation of the link, for the short-run index can continue until such time as a good link can be constructed. The first task should be to compute both the base-weighted (Laspeyres) form Q_{01} and the current-weighted (Paasche) form Q'_{01} of the index comparing years o and 1, using as many and as accurate series as possible. The comparison is confined to the years o and 1, ignoring all periods in between. Every effort should be made to include industries which have arisen since year o in the

computation of Q_{01} and industries which have disappeared prior to year 1 in the computation of Q'_{01} by using the method outlined in paragraph 92. Q_{01} and Q'_{01} are alternative estimates of the change in quantum of industrial production, i.e. of the link required. If their values show considerable divergence, then an investigation is needed of the reasons for the difference; important new products may have been introduced or there may have been changes on the side of consumers' demands or in the techniques and organization of production between the two years. The desirability of introducing an intermediate link between the years may be examined. On the other hand, if Q_{01} and Q'_{01} are not far apart in value, then the presumption is that a good link between years o and 1 can be established. Here the use of one of the "cross" forms of the index becomes relevant In preference to either Q_{01} or Q'_{01} some form such as Fisher's "ideal" formula $\sqrt{Q_{01} \times Q'_{01}}$ may be used as the link. In any case, there is not a unique solution to the problem. It is a matter of fixing the best link which can be devised in the light of all the evidence. Links of this sort may be long, e.g. between two years five or ten years apart, selected for a change of weight base, or they may be fixed between every consecutive pair of years. In the latter case, the links form a series of annual index numbers, either on a fixed base or chained together, and they will still need to be examined from time to time, to ensure that the longer link is good.

103.   In comparison with monthly or quarterly data, census data enable a better index to be computed because it can be shown with more and finer components and because it is possible to use more and better series derived from more extensive data and approximating more closely to work done in industry. First, census data make it possible to derive better output and input series, based on a larger selection of the products made or the materials used by an industry. Second, they enable more adequate adjustments to be applied to approximate series, for example an input series based on man-hours worked can be adjusted for changes in productivity of labour. Third, they permit the construction of series of net output at constant prices of products and materials, as described in section VI. This may well be the most profitable use of comprehensive data, apart from their use for reweighting. If such series can be constructed-and this is possible from census data though unlikely on a monthly basis-they will generally be much closer approximations to work done than output or input series.

104.   Relatively little experience has yet been obtained, except in a few countries, of the use of series of net output at constant prices. It is desirable that individual countries should experiment with such series on the basis of specially detailed data on output and materials used for particular industries. Important industries should be selected where it is expected that changes are taking place in the amount of processing applied to materials. It is not yet known how detailed the basic data should be for a reliable series of this kind, nor how much extraction and computational work is involved. Particular difficulties are that, since the series is derived as a difference between two gross valuations, the data on output and materials should correspond as closely as possible to the industry in questiong that the prices used should relate specifically to the industry and that output and materials should be interpreted widely to include unfinished goods on the one hand and "business" services on the other. Against these difficulties, there are certain advantages in the use of a difference of gross valuations. One is that

valuations of intermediate products passed from one establishment to another cancel out in computing the series. If the product of one sector in an industry is the material of another, e.g. flour passed on from the millers to the biscuit manufacturers, the valuation of product on the one side is equal to that of materials on the other and cancel in the difference (except for the cost of transport and handling between the miller and the biscuit maker, which is included in the valuation of the biscuit maker's materials but not of the miller's products; this cost also cancels of course because it is included in the valuation of the biscuits). Only the product not passed on, e.g. flour held by millers, need be valued and included.

105.   Consideration of the uses of census data to compute a comprehensive index for the setting-up of bench-marks raises the question whether the proposals should not go a little further, to make the bench-marks annual. There would be several advantages. An annual index would give a regular and frequent benchmark for the monthly index and it would enable the position to be reviewed, not only every five years, but every year. It would not be subject to such pressure for immediate compilation as the monthly index. The questions arising in adjusting approximate series included in the monthly index would either disappear (by restricting the basis of the monthly index) or be settled by the existence of corresponding but better series in the annual index. Examples are monthly series of output which need adjustment at intervals for incomplete coverage, and man-hour series which require correction for changes in productivity of labour. In some cases it would be possible to construct a supplementary annual index for small establishments which could be added to the index of production for large establish-ments, as envisaged in Section III above.

106.   However, the data needed for an annual index with a wide coverage are of the detailed kind provided in censuses or surveys of production. The prerequisite for its compilation is either an annual census of production or at least an extensive survey of production made annually. The practical question is whether the gain is sufficient to justify the additional work and expense involved. The additional work in its compilation would be off set in some part by savings in the calculation of the monthly index. For, with an annual index as a regular bench-mark, the monthly index could be confined to very short-run comparisons (monthly over a year or two) and it could be on a narrower basis, and compiled more quickly and cheaply, than would otherwise be necessary. Further, if the additional data needed for the annual index were already available, or if they would serve other useful purposes when collected, the net additional cost of compiling the annual index might be quite small.

107.   It is possible to compile a useful index of production, for month-to-month comparisons in the short-run, without taking a census of production. Further, it is possible to obtain at least rough links for use in longer-run comparisons provided that some extensive surveys of the field are made at intervals. The conclusion is inescapable, however, that the essential basis for a really satisfactory index for short-run and long-run comparisons is a regular and frequent (preferably annual) census of production of wide coverage. A frequent census would provide a regular test and review of a monthly or quarterly index of production, it would provide data for estimating relative weights whenever a change of weight base is required and it would be a means of devising better series, of adjusting monthly series and of solving most of

the problems of imputation inevitably encountered in a monthly index. Moreover, if the census is taken annually, all the testing, checking and revision can be undertaken every year. Finally, an annual census would support an annual index of production compiled with wide coverage to link together series of monthly index numbers computed quickly and promptly on a narrower basis.

108.   The proposals in Section IV are for a census of production taken every five years. It is now clear that there are many advantages in having a census more frequently and preferably annually, particularly for index numbers of production suitable for long-run comparisons. Countries, therefore, should generally aim at taking an annual census of production. If it is not feasible to take a full annual census, complete with data on labour and capital resources, as well as on net output, they should consider making an annual survey of industries with the particular object of obtaining data on the volume of output for constructing annual bench-marks on a broad basis. The annual survey might be restricted, not only by asking fewer questions, but by confining the coverage to large establishments defined, as envisaged in Section III, to include ninety percent of industrial production.

109.   It is concluded that bench-marks need to be set at intervals for linking separate short-period index numbers of production, that the links should be between particular years and that they should be estimated (e.g. by one of the "cross" forms of the index) in the light of both the base-weighted (Laspeyres) and the current-weighted (Paasche) forms. The essential basis,of a really satisfactory index of production, for long-run as well as short-run comparisons, is a frequent census of production and the aim should be to take a census more frequently than every five years. Preference should be given to an annual census on which could be based an annual index of production of wide coverage. The monthly index computed quickly and promptly on a narrower basis could then be linked to the annual index. If it is not possible to take a full annual census, a comprehensive annual survey of the volume of output in different industries should be made in inter-censal years.

X.   PRIORITIES

110.   In the previous sections, countries compiling or contemplating the compilation of an index of production are recommended to cover mining, manufacturing, utilities and construction; to show important industries separately in the framework on the Standard Industrial Classification; to weight by contributions to gross national product down to the level of major groups and preferably to census industries; to review and if necessary revise the weights at five-yearly intervals; to select or devise series which adequately represent work done, making special computations for industries where necessary; and to compute a primary index of production per working week together with one or more seasonally-adjusted indexes. A great deal of work is involved in the original design and regular compilation of an index of production of this sort, and further work is necessary on each revision. With only limited resources available, it is important to apply them so that they will yield the greatest accuracy in the index as regularly compiled. The first economy of resources, considered in this section, is to devote more effort to those parts of the index which will have greater effect on the result. A second method of eco-

nomizing, considered in the next section, is to use sampling methods instead of complete counts wherever this is profitable.

111.   Indications have been given in the previous sections of priorities within the subject matter of the section. It will be useful to draw these separate items together into one list. Detailed Planning will vary from country to country , but certain considerations can be set down which are of general application. One is that, beyond a point soon reached, attention is better paid to availability and accuracy of series than to elaboration of the weighting system. Unless there are enough current series available, which are also reasonably accurate, no amount of refinement of weights will produce an accurate index; while, subject to certain provisos about correlation between weights and series, moderate variations in weights may not greatly affect the index, in total or in the main groups. It is important to set accurately the main calibration of weights by good estimates of net output (contribution to gross national product) for the broad industrial categories; but further calibration can be made more roughly and additional work on weights can be left to be done if resources permit. Another consideration is that attention should always be concentrated on large industries and on large sectors within industries. Smaller sectors can be given less attention than larger; they can be represented by fewer or more approximate series. Little is gained if a small sector of an industry is represented by several accurate series while the rest of the industry is dependent on one inaccurate series. Another point is that while it is important that the broad lines of the classification system adopted should be internationally comparable, too much effort should not be devoted to transferring very small items from one group to another in the attempt to make the classification conform exactly to the international standard.

112.   In the light of these considerations, a list of priorities in general terms can be drawn up. This list is not necessarily applicable in all respects to the problems of any one country; but it will serve to illustrate the relative importance that may be attached to different operations in the normal case. First priority should be given to:

(a) Selecting or devising current series to represent work done in mining, manufacturing and utilities, regularly correcting such series in the light of broader-based annual or other data and constructing a supplementary index at census intervals for small establishments. The aim should be to cover not more than ten per cent of production in the supplementaxy index. Of the remaining ninety per cent, representing large establishments, part will be covered by direct returns and part by imputation, but the establishments included in the monthly or quarterly index by means of direct returns should account for two-thirds or more of the net output of large establishments. Very careful atten-tion should be paid to industries carrying a weight of three per cent or more.
(b) Fixing accurate weights, proportional to either contribution to gross national product or census value added, for divisions (1-digit), major groups (2-digit) and census industries (3-digit).
(c) Analyzing the total for mining, manufacturing and utilities to show separately the three divisions (1-digit) and, within these divisions, major groups (2-digit) which are important in the country concerned.

(d) Ensuring that these divisions and major groups correspond sufficiently closely to the international classification to be reasonably comparable internationally.

(e) Computing the series on a working week basis.

(f) Reviewing the whole index every five years.

Second priority should be given to:

(a) Compiling an index for construction.

(b) Increasing the proportion of large establishments included in the monthly or quarterly index by means of direct returns so that
four -fifths or more of the net, output of these establishments is covered without the use of imputation.

(c) Further analyzing the total to show important census industries (3-digit) and less important major groups (2-digit) separately.

(d) Compiling estimates of weights representing contribution to gross national product, as distinct from census value added, for divisions (1-digit)

(e) Further examining and selecting between different current series to represent work done in industries carrying weights between 1 per cent and 3 per cent.

Third priority should be given to:

(a) Deriving secondary series with seasonal variations eliminated. In many cases this will not be possible until a sufficiently long run of post-war figures becomes available to permit the computation of an accurate index of seasonal variations. The minimum length of run required for this purpose will vary from country to country depending upon the type of seasonal variation displayed by the series and the degree of stability of the seasonal pattern. The determination of the minimum run required can best be left to the judgment of the compilers of each national index.

(b) Securing direct coverage, monthly or quarterly, of at least two-thirds of the output of large establishments for each division (1-digit) and for each major group (2-digit) shown separately.

(c) Further analyzing the total to show less important census industries (3-digit) separately.

(d) Compiling estimates of weights representing contribution to gross national product, as distinct from census value added, for major groups (2-digit) and census industries (3-digit).

(e) Selecting between different bases of weighting for sub-divisions of census industries (3-digit).

(f) Further examining and selecting between different current series to represent work done in industries carrying weights of under 1 per cent.

XI. THE USE OF SAMPLING METHODS

113.   The second way of economizing resources is by the use of sampling methods. Countries should consider carefully the various ways in which these methods might be applied in the construction of an index. The following are the fields in which their use appears most promising, but all parts of the enquiry should be examined since the area of application of modern techniques is potentially very wide.

114.   Sample surveys can be used in several ways to supplement regular censuses of production, and thereby assist in the derivation of weights:
   (a) Virtually all countries exclude small establishments from their censuses, the limit of size being usually defined by the number of workers employed or the gross value of output. In many countries small establishments account for a significant part of total industrial production and in some industries their share is substantial. The census data can be greatly improved in comprehensiveness by collecting returns from a sample of these small establishments.
   (b) Countries which find it too expensive or too difficult to collect data on supplementary costs (expenditures for advertising, insurance, stationery, postage, telephone and telegraph services, legal and accounting services, etc.) from all establishments could collect such information from a sample in each census industry. This would enable them to make reliable estimates of the relation between contribution to gross national product and census value added for each census industry and thus to derive weights proportional to contribution to gross national product.
   (c) In the same way countries which are unable to undertake the collection from all establishments of data on the cost of raw materials, containers, fuel, purchased electric power and contract work performed by other establishments might employ sampling methods to secure this basic information. This would permit the derivation of weights proportional to census value added from data on gross value of output.

115.   Sample censuses of production might be conducted in inter-censal years by countries which do not take an annual census. The resulting data could be used to construct an annual index of production or to improve the accuracy of an existing annual index. These sample censuses would have the additional advantage of assisting the census authorities in keeping their register of establishments up-to-date. If comprehensive sample censuses of production are considered too expensive, annual or biennial sample enquiries might be made during inter-censal periods in important industries for which man-hours worked or consumption of raw materials are used as measures of current changes in work done, with the object of correcting the current series used, to allow for changes in productivity or economies in the use of raw materials. Special sample enquiries can be used to collect data required in estimating weights for important new industries which have arisen since the last complete census. In some cases this would make it possible to include the new industry in the existing index on a provisional basis pending a complete review (see paragraph 92).

116.   Sample enquiries can be used to supplement (when necessary) the data on current production. In many countries figures on monthly production are collected only from larger establishments and they could be made more reliable by sample returns from the smaller establishments collected at (e.g.) three or six monthly intervals. In some

industries current production data might be collected entirely by sample, particularly from industries in which there is a large number of small, independent establishments.

117.   There is however, a serious difficulty in adopting sampling methods to collect data on changes in economic variables, as envisaged in paragraphs 115 and 116, as opposed to one-time enquiries into absolute values, as envisaged in paragraph 114. Unless the sample is made large, and therefore expensive, the change from one period to the next may well be within the limit of the sampling error and will not be detected. This is more likely when the variable measured, e.g. the level of output of an industry, is relatively stable. The cases in which sampling methods can be used have therefore to be carefully selected and in general are confined to industries where changes are considerable.

118.   Sample enquiries can be employed in collecting data for various research purposes. For instance, data might be secured for use in experiments to determine the relative merits of alternative types of series representing work done or of alternative bases of internal weighting for a particular industry.

119.   The sampling technique chosen will depend on the availability of lists of industrial establishments. The following suggestions may be found useful. In countries where the lists of establishments are reasonably complete, a design of stratified sampling with variable sampling fractions for different strata should be considered. Where the lists are not complete, as is likely to be the case if there are large numbers of small establishments, different designs may be adopted for the large and small establishments. For large establishments a complete list may be prepared and stratified sampling may be used, with variable sampling fractions as above. For small establishments a complete list may be prepared for the principal towns, and samples drawn from the list stratified by regions, with uniform or variable sampling fractions. Small establishments in rural areas and in small towns can only be covered, within reasonable costs, by some form of area sampling.

120.   The application of modern sampling methods to statistics of industrial production is as yet in a very early stage of development, and little experience has been gained in dealing with the problems raised by different types of enquiries. In view of the potential importance of these methods, however, countries are urged to investigate the possibilities carefully. The results of these investigations and experiments will provide the necessary material for a fuller study of the problem and a better evaluation of the ways in which sampling methods can be most profitably employed in this field.

XII. RELATION TO OTHER GENERAL ECONOMIC INDICATORS

121.   The relation of the index of industrial production to national income and product has been considered in section II. In this section the relation is examined between the index and three other general economic indicators, namely the wholesale price index, the level of employment and the quantum of exports. It would clearly be very useful for economic analysis if movements in the index of industrial production could be compared with movements in these other indicators. In practice there may be differences due to the fact that the index of production relates only to industry, while the other three indicators relate to the whole economy, i.e., total employment, general wholesale prices and total exports or imports. These differences are not considered here. The comparison which is made is between the production index and the corresponding parts of the other indicators, i.e., the field of industry including materials and semi-finished products and excluding agriculture, transport and services.

122.   Relation to the wholesale price index. The index of production differs from the wholesale price index, as this is generally constructed, in three main respects. Firstly, the production index is based on establishments grouped into industries, while the price index is usually based on commodities according to kind with only occasionally an industrial sub-grouping. This means that the contents of similarly named groups in the two sets of index numbers are not likely to be strictly comparable. The basic items of a price index are chosen as representative of groups of items which are materially similar, while in the index of production the items are representative of groups of items which are produced in similar establishments. Secondly, in the case of the total index numbers the coverage may differ substantially since the index of production covers home-produced goods, while the price index usually includes also imported goods. Thirdly, the index of production is weighted by net values and the price index generally by gross values. The weights given to different goods will therefore vary in the two indexes. Also, the current production series will not (or should not) reflect a change in import content of the product while the current price series will.

123.   Strict comparability between the index of production and an index of wholesale prices would therefore require the construction of a special price index. This index would be confined to home-produced goods, classified industry-wise instead of commodity wise, weighted net instead of gross and would use series derived from price margins instead of gross prices. The wholesale price index as generally constructed is therefore not comparable with the production index and great caution should be exercised in relating one to the other. A price index of special construction, difficult to realize in practice, is needed to be comparable with the production index. An alternative would be to compare an index of deliveries (as opposed to production) with an index of the wholesale prices (gross) of final goods.

124.   In addition to comparisons between the total indexes, the aggregative form of the index of production computed as the difference between gross valuations of products and of materials (see appendix D) can be used in conjunction with sepa wholesale price indexes of home-produced products and materials. Thus the numerator of the equation

$$Q_{01} = \frac{\Sigma P_0 Q_1 - \Sigma \pi_0 \mu_1}{\Sigma n_0}$$

could be derived as

$$\frac{\dfrac{\Sigma P_1 Q_1}{\Sigma P_1 Q_1}}{\Sigma P_0 q_1} - \frac{\dfrac{\Sigma \pi_1 \mu_1}{\Sigma \pi_1 \mu_1}}{\Sigma \pi_0 \mu_1}$$

In other words, the value of current output, and of materials used, could be deflated by indexes of (gross) wholesale prices confined to home-produced output, and materials respectively.

125. <u>Relation to the quantum of export.</u> An index of the quantum of exports, like a wholesale price index, is based on commodities and therefore will probably not correspond, in its parts, with the similarly named parts of the production index. With regard to the scope, the export index will be comparable with the production index provided that it excludes re-exports. As generally constructed, however, the export index is in gross terms, i.e., the exports are valued at constant gross prices and not at constant price margins. Therefore, the export index does not indicate changes in the volume of that part of domestic output which is exported. For instance, if there is an increase in the import content of exports, the export index can increase without any change occurring in the amount of domestic work done on goods for export. For strict comparability with the production index, therefore, the index of exports should be reweighted with net outputs as weights and classified industry-wise instead of commodity-wise.

126. <u>Relation to the index of employment</u>. Like the production index, the employment index is based on establishments grouped into industries, it refers to domestic production and it relates to net output, not gross output. If both are satisfactorily constructed, therefore, the index of production can be related to the, index of employment to yield an index of productivity. The one index measures the result of the input of all factors of production, the other measures the input of one factor. It should be noted, however, that the errors of the two index numbers may be compounded in this process with the result that the error of the index of productivity so obtained may be greater than the actual changes in productivity since productivity normally fluctuates within very narrow limits.

APPENDIX A

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The International Standard Industrial Classification of all Economic Activities

List of divisions and major groups to be covered by the index of industrial production

<u>Division</u>	<u>Major Group</u>
1. Mining and quarrying	11. Coal mining
	12. Metal mining
	13. Crude petroleum and natural gas
	14. Stone quarrying, clay and sand pits
	19. Non-metallic mining and quarrying not elsewhere classified
2-3. Manufacturing	20. Food manufacturing industries, except beverage industries
	21. Beverage industries
	22. Tobacco manufactures
	23. Manufacture of textiles
	24. Manufacture of footwear, other wearing apparel and made-up textile goods
	25. Manufacture of wood and cork, except manufacture of furniture
	26. Manufacture of furniture and fixtures
	27. Manufacture of paper and paper products
	28. Printing, publishing and allied industries
	29. Manufacture of leather and leather products, except footwear
	30. Manufacture of rubber products
	31. Manufacture of chemicals and chemical products
	32. Manufacture of products of petroleum and coal
	33. Manufacture of non-metallic mineral products, except products of petroleum and coal
	34. Basic metal industries
	35. Manufacture of metal products, except machinery and transport equipment
	36. Manufacture of machinery, except electrical machinery
	37. Manufacture of electrical machinery, apparatus, appliances and supplies
	38. Manufacture of transport equipment
	39. Miscellaneous manufacturing industries
4. Construction	40. Construction
5. Electricity, gas, water and sanitary services	51. Electricity, gas and steam

Note on the measurement of an industry's contribution to
gross national product

The contribution of an industry to gross national product (at factor cost) may be defined as the unduplicated aggregate value of the goods and services produced in the industry during a period of time including the increase in the value of work in progress, less the value of the goods and services bought from other industries and used up in the process of production. The contribution to net national product is obtained if the cost of maintaining capital intact is also subtracted. The precise implications of the definition are discussed in technical treatises on the concepts of national income and gross national product. The following list of items that must be deducted from the gross value of output of an industry to obtain its contribution to gross national product (at factor cost) may be considered sufficient for practical purposes.

1. Costs of materials and fuels used and of work given out.
2. Printing stationery and other office supplies.
3. Advertising and other selling expenses.
4. Business insurance premiums.
5. Postage, telegraph and telephone payments.
6. Expenses for banking, legal, accounting, auditing, and similar business services.
7. Cost of small repairs, maintenance and servicing carried out by outside contractors.
8. Cost of materials and parts required for small repairs and maintenance.
9. Property taxes and water rates.
10. Excise and sales taxes if these have not already been deducted from the gross value of production (less subsidies, if any, received from the Government).

Items 3 and 6 refer only to outlay on services bought from other industries.

The raw materials and services bought from other enterprises should be valued according to the same principles as the goods and services produced. If the contribution to gross national product is to be expressed in current prices, both the gross value of output and the cost of raw materials and services should be valued on that basis to avoid inclusion of capital gains or losses due to price fluctuations.

With respect to certain items of expenditure it is necessary to decide whether they should be considered as current business costs or as outlay on capital goods. In the former case, the item should be included in the above list of deductions. There are various definitions possible of indirect taxes and of government subsidies; but the differences are in general slight. For a discussion

of these and similar problems reference is made to the technical literature on national income and capital formation.

The contribution of an industry to gross national product is equal to the aggregate income paid to the factors of production which contributed to the productive process plus the cost of capital used up. Therefore, it is equal to the total of the following items:

1. (a) Wages and salaries
    (b) Other labour income
2. Income of unincorporated enterprises
3. Corporate profits before taxes
4. Net interest
5. Net rents on lands and buildings, including royalties
6. Allowances for depreciation and obsolescence.

Wages and salaries include employees' contributions to social insurance and pension funds, income in kind, commissions, tips, bonuses, etc. Other labour income includes employers' contributions to social insurance and pension funds, pensions and compensations paid if no social security and pension funds exist, etc. Income of unincorporated enterprises and corporate profits exclude capital gains and losses, and dividends received from other enterprises. Corporate profits, allowances for depreciation and obsolescence and other items will have to be defined in accordance with the detailed treatment of various items in the first list. Rent paid on lands and buildings might be considered as part of income originating in the real estate industry. If this treatment is adopted, the item should be included in the list of items to be deducted from the gross value of output.

APPENDIX D

Technical notes

1.   Notation.   For each product in the index of production, denote:

Quantities	Prices

Q = output of final product

q = amount of work done

Φ = amount of materials used price

P = price of product = selling value per unit of product

p = price margin = value added per unit of work done

π = price of materials = buying price

per unit of materials

Further, denote:

m = man-hours worked corresponding to work-done q

n = pq = value of net output

$k = \dfrac{q}{Q}$ = work-done content per unit of product

$K = \dfrac{\mu}{Q}$ = materials content per unit of product

$v = \dfrac{q}{m}$ = product (work-done) per man-hour

$w = \dfrac{1}{v} = \dfrac{m}{q}$ = man-hours per unit of product (work-done)

Then the base-weighted and current-weighted index numbers of production, with values of net output as weights, are

$$Q_{o1} = \frac{\Sigma p_o q_1}{\Sigma p_o q_o} = \frac{\Sigma n_o(\dfrac{q^1}{q_o})}{\Sigma n_o}$$

$$Q'_{o1} = \frac{\Sigma p_1 q_1}{\Sigma p_1 q_o} = \frac{1}{\dfrac{\Sigma n_1(\dfrac{q_o}{q^1})}{\Sigma n_1}}$$

and with man-hour weights

$$\overline{Q}_{o1} = \frac{\Sigma w_o q_1}{\Sigma w_o q_o} = \frac{\Sigma m_o (\frac{q^1}{qo})}{\Sigma m_o}$$

$$\overline{Q}_{o1'} = \frac{\Sigma w_1 q_1}{\Sigma w_1 q_o} = \frac{1}{\frac{\Sigma m_1(\frac{qo}{q^1})}{\Sigma m_1}}$$

where the periods compared are denoted by suffixes o and 1 and where Σ extends
over all products in the index.

2.   The definition of work-done series.   For a single product produced from a single raw material, the value of net output in any period is

$$n = PQ - \pi \mu$$

We need to split n into the product p x q where q represents work done and p the corresponding price (i.e. price margin per unit of work done). The price (p) could then be held constant to isolate changes in work done (q) over time. This split is possible as a concept, but it cannot be defined statistically. This is true even in the simplest case of one product and one material here considered. There are two possibilities, one being to define

$$p_o q_1 \quad = \quad P_o Q_1 - \pi_o \mu_1$$

to give $$\frac{q_1}{q_o} = \frac{PoQ_1 - \pi o \mu_1}{PoQo - \pi o \mu o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(1)
and the other to define

$$p_1 q_o \quad = \quad P_1 Q_o - \pi_1 \mu_o$$

to give $$\frac{q_1}{q_o} = \frac{P_1 Q_1 - \pi_1 \mu_1}{P_1 Qo - \pi_1 \mu o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(2)

In general, (1) is not equal to (2). Let $Q = \lambda_o \mu$ in period o and $Q = \lambda_1 \mu$ in Period 1, λ_o and λ_1 being the input-output (technical) relations. Then

from (1) $$\frac{q1}{qo} = \frac{\mu1}{\mu o} \cdot \frac{\lambda1 Po - \pi o}{\lambda o Po - \pi o}$$

and from (2) $$\frac{q1}{qo} = \frac{\mu1}{\mu o} \cdot \frac{\lambda1 P1 - \pi1}{\lambda o P1 - \pi1}$$

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which are equal if

$$\frac{\lambda 1 - \dfrac{\pi o}{Po}}{\lambda o - \dfrac{\pi o}{Po}} = \frac{\lambda 1 - \dfrac{\pi 1}{P1}}{\lambda o - \dfrac{\pi 1}{Po}}$$

This is true only if $\lambda_o = \lambda_1$ or if $\dfrac{\pi o}{Po} \bullet \dfrac{\pi_1}{P_1}$. Hence (1) and (2) are not equal

provided that the technical input-output relation changes ($\lambda o \neq \lambda_1$) and the

prices of material and product do not change in proportion ($\dfrac{\pi o}{Po} \neq \dfrac{\pi_1}{P_1}$). It

follows that we cannot define p and q statistically; or, what comes to the same thing, we cannot write poq1 = $P_o Q_1$ - πoμ1 and $p_1$qo = $P_1 Q_o$ - $\pi_1 \mu$o.

In practice, the industries for which net output (n) is given are such that in each several materials (including fuels and business services) are transformed into several products. Work in progress, as represented by intermediate products, must be taken into account (see section VI, paragraph 50). It is possible to allow for variations in types of products and materials by including separate data on each type. The difficulties arise because of changing work in progress and because of changes in the amount of processing applied to materials (i.e. changing technical input-output relations). The latter difficulty arises even if one material is transformed into one product and becomes more serious in the general case.

3.   Changes in amount of processing. To isolate the second of the two difficulties, assume that work in progress remains unchanged from one period to another. The final index numbers of production, *Qo*1 or *Q'*o1 are to be obtained by aggregating data for the separate (census) industries. The problem is to get, for each such industry, an estimate of q_{o1}, the change in

work done (i.e. what would be written if $\dfrac{q1}{qo}$ if q could be defined

statistically). Write Σ for the sum over all products or all materials in the industry. The net output in any period

$$n = \Sigma pq = \Sigma PQ - \Sigma \pi \mu$$

The work-done series is defined conceptually, but not statistically, as

$$q_{o1} = \frac{\Sigma p_o q_1}{\Sigma p_o q_o} \qquad \text{or} \qquad q'_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_1 q_o}$$

$$\dots\dots\dots\dots\dots .3$$

If there is one product and one material, q_{o1} = q'_{01} = $\dfrac{q_1}{q_o}$; while qo1 and q'_{01}

differ when there are several products and materials (the familiar index-number case).

As statistical measures, we might suggest

$$q_{o1} = \frac{\Sigma P_o Q_1 - \Sigma \pi_o \mu_1}{\Sigma P_o Q_o - \Sigma \pi_o \mu_o} \quad \text{or} \quad q'_{o1} = \frac{\Sigma P_1 Q_1 - \Sigma \pi_1 \mu_1}{\Sigma P_1 Q_o - \Sigma \pi_1 \mu_o}$$

.....................4

Then q_{o1} cannot be taken equal to q'_{o1} even when there is only one product and one material. So q_{o1} and q'_{o1} cannot be more than approximations to work-done series.

Now, if all materials going into one product are lumped together for convenience, so that $k = \dfrac{q}{Q}$ and $K = \dfrac{\mu}{Q}$ can be taken as the work-done and materials content (per unit of product), then

$$PQ = pq + \pi\mu$$
or
$$P = kp + K\pi$$

The proportions k and K can vary from one period to another; for instance, if more work is done on less fabricated materials for a given product, then k increases while K decreases. So

$$\Sigma P_o Q_1 - \Sigma \pi_o \mu_1 = \Sigma(P_o - K_1 \pi_o) Q_1$$
$$= \Sigma(k_o p_o + K_o \pi_o - K_1 \pi_o) \frac{q1}{k_1}$$
$$= \Sigma p_o q_1 - \Sigma \frac{q_1}{k_1} \{ (k_1 p_o + \kappa \pi_o) - (k_o p_o + \kappa_o \pi_1) \}$$
$$= \Sigma p_o q_1$$

if $P_o = k_o p_o + K_o \pi_o = k_1 p_o + K_1 \pi_o$ for each product ... (5). Hence q_{o1} from (4) approximates to the conceptual work-done series (3), if the condition (5) is approximately satisfied. There must, in short, be compensating variations in k and K, the work-done and materials contents of each product. In practice, with the technical input-output relations changing, k and K will vary in opposite directions so that (5) is at least very approximately satisfied.

A similar argument holds for q'_{o1} in form (4). The general conclusion, then, is that no statistical definition of q_{o1} or q'_{o1} in form (3) is possible. But either form (4) will generally serve as a good approximation to the concept and these are statistically defined. Some allowance, at least, is made in (4) for changes in amount of processing.

4. Changes in work in progress. If we assume, more generally, that work in progress changes from one period to another, then some modifications are needed. It is no longer true that $n = \Sigma PQ - \Sigma \pi\mu$. Indeed, the difference could be

negative if there are extensive additions to work in progress. Hence the forms (4) need to be adjusted if they are to remain approximations to work-done series. This is to be done by extending the sum ΣPQ (e.g. $\Sigma P_o Q_1$ or $\Sigma P_1 Q_o$) to include intermediate products as well as final products. This implies the definition of appropriate prices of intermediate products. These intermediate products may appear both as products and as materials used within an industry and so they cancel out in large part. But they do not cancel out completely when work in progress is changing within the industry as a whole.

5. <u>Conclusions on definition of work-done series</u>. To obtain index numbers of the form Q_{o1} or Q'_{o1}, data relating to separate (census) industries are aggregated and a measure of changes in work done in each industry is required. Conceptually, the measure would be q_{01} or $q`_{01}$ given by (3). These expressions, however, cannot be defined statistically and some approximation to them must be sought. The nearest statistical measure will generally be of the form q_{o1} or q'_{01} given by (4). Such a measure can be made to allow for differentiation in product and in materials used and for changes in the amount of work in progress. It also allows, at least in part, for changing amount of processing applied to materials, i.e. for variations in the technical input-output relations. However, the measure does not allow completely for such variations as is seen from the fact that q_{o1} is not equal to q'_{01}, as given by (4), even in the simplest case of one product and one material (a case which avoids the index-number problem arising in aggregates).

The forms (4) have one big advantage in practice. They do not depend on the particular grouping of industries that may be adopted. They can be aggregated directly for any group of industries or for all industries, provided that the net output weights of Q_{o1} or Q'_{o1} are used. Product which appear as the output of one industry and as the material of another industry within the group cancel out in the summation process.

Exactly similar conclusion apply to the alternative index number of proudction, $\overline{Q}_{o1}$ or $\overline{Q}'_{o1}$, which use man-hour weights. All that is necessary to replace the prices (p) with man-hours per unit(w).

6.   <u>Measurements of productivity of labour</u>. For a given prdouct or sector of an industry, productivity of labour is to be measured by v=product (work-done) per man-hour, or by the reciprocal of w=man-hours per unit of product (work done).   In practice, product is generally replaced by gross output but this introudces an approximation of the same type as that involved in using an output series for work-done in an index of production.   The problem is to define a measure of change in productivity over the whole field of industrial production, and this is a problem of index numbers.   Let I denote such an index of productivity change from period o to period 1.

A first and direct measure for I is the division of an index of production by the change in total man-hours worked over all industry. Take $Q_{o1} = \dfrac{\Sigma p_o q_1}{\Sigma p_o q_o}$ as the index or production, using the base-weighted form with net output weights. Then this is to be divided by $m_{o1} = \dfrac{\Sigma m_1}{\Sigma m_o}$

to give

$$I = \frac{Q_{o1}}{m_{o1}} = \frac{\dfrac{\Sigma p_o q_1}{\Sigma m_1}}{\dfrac{\Sigma p_o q_o}{\Sigma m_o}}$$

as the change in average product per man-hour over all industry. I is an <u>average</u> concept and msut be interpreted as such. It allows for all shifts in activity as between high and low productivity indsutries as well as for changes in productivity within industries.   For example, I could decrease even though productivity rose in every industry, if activity shifted to industries with low proudctivity. I would be related, in fact, to average earnings of workers rather than to wage rates.

For some purposes, it is desirable to separate the two factors which affect I, i.e. the effect of shifting activity between industries and the effect of productivity changes wthin industries. The latter factor may be the one sought as an index of changes in productivity of labour. In this case, further analysis of I is needed and this can be done in two ways.

(1) Start with Q_{o1} and write $v^1 = p_o v = $ valuation of product per man-hour (at base prices p_o)

Then $Q_{o1} = \dfrac{\dfrac{\Sigma p_o v_1 m_1}{\Sigma p_o v_o m_o} = \dfrac{\Sigma v_1^1 m1}{\Sigma v_o^1 mo}}{}$

which either $\quad = \dfrac{\Sigma v_o^1 m_1}{\Sigma v_o^1 m_o} \times \dfrac{\Sigma v_1^1 m_1}{\Sigma v_o^1 m_1}$

or $\quad = \dfrac{\Sigma v_1^1 m_1}{\Sigma v_1^1 m_o} \times \dfrac{\Sigma v_1^1 m_o}{\Sigma v_o^1 m_o}$

Q_{o1} is split into two components: the first is an index of the effect of changing activity (man-hours, m) in different sectors, and the second is the required index of productivity changes (weighted with man-hours, m_o or m_1). Hence, alternative index numberss of productivity emerge:

$I_1 \quad = \quad \dfrac{\Sigma v_1^1 m_1}{\Sigma v_o^1 m_1} = \dfrac{Q_{o1}}{\overline{m}_{o1}} \quad$ where $\quad \overline{m}_{o1} = \dfrac{\Sigma v_o^1 m_1}{\Sigma v_o^1 m_o}$

$I_2 \quad = \quad \dfrac{\Sigma v_1^1 m_1}{\Sigma v_o^1 m_1} = \dfrac{Q_{o1}}{\overline{m}{}^1_{o1}} \quad$ where $\quad \overline{m}{}^1_{o1} = \dfrac{\Sigma v_1^1 m_1}{\Sigma v_o^1 m_o}$

In each of these forms, the index of productivity is derived by dividing Q_{o1} by a corrected index of man-hours worked, m_{o1} or $m^1{}_{o1}$, instead of by the crude index of man-hours worked m_{o1}. The productivity measure here is product per man-hour, valued at fixed (base) prices of products. Two further index numbers, I3 and I4, could be derived by starting with $Q^1_o{}_1$, in which case productivity is valued at fixed (current) prices of prodcuts.

(2) $\qquad$ (2) Start with m_{o1} and write:

$$m_{o1} = \dfrac{\Sigma m_1}{\Sigma m_o} = \dfrac{\Sigma w_1 q_1}{\Sigma w_o q_o}$$

which either $\quad = \dfrac{\Sigma w_1 q_1}{\Sigma w_1 q_o} \times \dfrac{\Sigma w_1 q_o}{\Sigma w o q_o}$

or $\quad = \dfrac{\Sigma w o q_1}{\Sigma w o q_o} \times \dfrac{\Sigma w_1 q1}{\Sigma w o q1}$

m_{o1} is split into two components; the first is an index of the effect of changing activity (production, q) and the second is an index of changes in man-hours per unit of product (weighted with production, qo or q1). The latter is the recipriocal of the required index of productivity changes and so alternative forms emerge again:

$$I^1_1 = \dfrac{1}{\dfrac{\Sigma w_1 q_1}{\Sigma w_o q_1}} = \dfrac{\Sigma w_o q_1}{\dfrac{\Sigma w_o q_o}{m_{o1}}} = \dfrac{\overline{Q}_{o1}}{m_{o1}}$$

and a similar form I^1_2 which includes $Q_o{}^1_1$ instead of Q_{o1}. Here, the index or productivity is the result of dividing the index of man-hours worked (m_{o1}) into an index

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of production with man-hour (and not net output) weights. The productivity measure
now is the reciprocal of man-hours per unit of product.

7. <u>Conclusions on measures of productivity of labour.</u> If productivity of labour is used in the sense of product aggregated over all industry per man-hour worked, then the appropriate measure of changing productivity is

$$I = \frac{Qo1}{mo1}$$

where an index of production (weighted with values of net output) is divided by changes in total man-hours worked. Such a measure will show the effects both of shifting activity between industries and of productivity changes within industry. On the other hand, if the concept of productivity relates only to the second of these two factors, then various index-number constructions can be used in its measure. There are two broad approaches. In the first, the basic concept is product per man-hour combined by valuing at fixed product prices. The measure of changing productivity which results is of the form:

$$I_1 = \frac{Qo1}{mo1}$$

where the index of production Qo1 remains while the index of man-hours worked is adjusted (to $\overline{m}_{o1}$) by weighting with value of product per man-hour. In the second approach, the concept is man-hours per unit of product (the reciprocal of which is productivity of labour) and the measure of changing productivity is of the form:

$$I_1^1 = \frac{\overline{Q}o1}{mo1}$$

where the index of production is modified (to $\overline{Q}o1$) by weighting with man-hour weights while the figures of total man-hours worked (in mo1) are unchanged. The choice between these alternative forms of index numbers is mainly a matter of purpose and convenience.

8. <u>Comparison base.</u> Let period 0 be the weight base and period 1 the comparison base, with 2 denoting any current period. Then, in the original series of index numbers on period 0 as comparison and weight base, two items are:

$$Q_{o1} = \frac{\Sigma poq1}{\Sigma poqo} = \frac{\Sigma no(\frac{q1}{qo})}{\Sigma no} = \quad \text{and} \quad Q_{o2} = \frac{\Sigma poq2}{\Sigma poqo} = \frac{\Sigma no(\frac{q2}{qo})}{no}$$

To express production in period 2 on period 1 as a comparison base (weighting as in period 0), we compute

$$Q_{12(o)} = \frac{Qo2}{Qo1}$$

This is the definition of the series on a comparison base different from the weight base; the series on the weight base as 100 is simply divided through by the figure for the period chosen as comparison base.

There are two methods of computing $Q_{12(o)}$ without first obtaining index numbers on period 0.

(1) Valuation of quantities q in the periods (1 and 2) compared with prices determined in the weight base period (po), i.e.

$$Q_{12(o)} = \frac{\Sigma poq2}{\Sigma poq1}$$

which is equivalent to $\dfrac{Qo2}{Qo1}$

(2) Average of quantity relatives ($\dfrac{q2}{q1}$) on the comparison base, i.e.

$$Q_{12(o)} = \frac{\Sigma w1(\frac{q2}{q1})}{\Sigma w1} \qquad \text{where } w_1 = p_oq_1$$

which is again equivalent to $\dfrac{Qo2}{Qo1}$. The important point here is that the weights w_1 are not values of net output (n_o) in the weight base period, but adjustments of these values. The adjustment involves taking quantity changes ($\dfrac{q1}{qo}$) from weight base to comparison base, and applying them to the net outputs (n_o) in the weight base:

$$w_1 = p_oq_1 = n_o \frac{q1}{qo}$$

Once these adjusted weights are fixed for any selected comparison base, the index on the comparison base can be computed without reference to the weight base.