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Price Stability in a Monetary Union

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Abstract

From the fiscal and monetary criteria indicated in the Maastricht Treaty, it appears that its architects were concerned with the risks of price instability and excessive public debt within the European Monetary Union. The goal of price stability is unanimously regarded as a solid principle for the success of the union. Several criticisms have been voiced over the years against the fiscal criteria. It is not clear, firstly, whether fiscal coordination is at all necessary for price stability but, secondly, also whether the fiscal criteria of the Treaty are best suited for this purpose. The paper analyzes whether coordination of fiscal policies within the union is necessary for price stability. The analysis is conducted using a standard general equilibrium model representing a monetary union with two countries featuring monopolistic competition and price rigidity. It turns out that some form of fiscal coordination is required for equilibria in which monetary policy alone is able to secure price stability.

Key words: Monetary Union, Monetary Policy, Fiscal Policy,Fiscal Coordination

JEL classification: E63

1 Introduction

From the fiscal and monetary criteria indicated in the Maastricht Treaty, it appears that its architects were particularly concerned with the risks of price instability and excessive public debt within the European Monetary Union. In particular, in order to prevent excessive and unsustainable public debts, the fiscal criteria include ceilings to the ratio deficit to GDP and to the ratio debt to GDP by respectively three and 60 percent, along with a no bail-out commitment on the part of the member governments and of the common Central Bank on behalf of members experiencing fiscal imbalances. The monetary criterion, on the other hand, indicates a sole mandate for the common central bank: price stability within the union. Specifically, a quantitative target has been specified in terms of a weighted average of the harmonized index of consumer-price of the countries belonging to the union.

Although the goal of price stability within the union is almost unanimously regarded as a solid principle for the success of the union, several criticisms have been voiced over the years against the fiscal criteria. It is generally agreed that the individual fiscal authorities of the countries in the union should conduct a sound fiscal policy. It is not clear, however, whether the fiscal criteria of the Treaty are best suited for this purpose. Indeed, when the Maastricht Treaty was concluded most of the European countries did not meet one or both criteria, and in light of several studies, only the debt of Italy and Greece were considered as not sustainable. The main criticism against such fiscal criteria is that they do not allow for countercyclical fiscal policy, thus being too though on countries experiencing fiscal distress and conceivably prolonging the pain of a recession.

In light of the above, the purpose of the paper will be to analyze whether coordination of fiscal policies within the union is necessary for price stability and to offer indications on what form of coordination could be pursued, accordingly.

The analysis is conducted using a fairly standard general equilibrium model representing a monetary union with two countries. The framework that we propose is a New Keynesian model featuring monopolistic competition and price sluggishness[1]. We consider the simplest scenario: that of a monetary union composed by two identical countries subjected to heterogeneous shocks. It will turn out that monetary policy alone can determine equilibrium prices and quantities within the union. A unique and bounded equilibrium exists only when the individual fiscal authorities act responsibly. In such an equilibrium, monetary policy succeeds in maintaining price stability within the union. To test the robustness of such results to a more elaborated model, we allow countries to be heterogeneous. It turns out that the previously mentioned result is robust to more complex specifications.

However, the equilibrium may fail to exist when individual fiscal authorities do not respond to the monetary policy in a responsible way. In such cases, governments' debt would not find its markets - as Ricardian equivalence would fail to hold - at the equilibrium prices determined by the sole monetary authority. The fiscally irresponsible countries will eventually be forced to default or to abandon the union.

In addition, a unique and bounded equilibrium may exist when the monetary authority cannot credibly commit itself to a rule and the fiscal authorities act as leaders. In such an equilibrium, the level of prices depends on the level of governments' debt. Monetary policy alone can no longer guarantee price stability within the union.

Therefore, some form of fiscal coordinations seems to be necessary for the objective of price stability.

The paper is organized as follows: section 2 develops the baseline model and analyzes the issue of determinacy of the equilibrium. Section 3 derives the equilibrium under active monetary policy. Section 4 analyzes the equilibrium with active fiscal policy and conducts a simulation of the consequences of shocks to government primary deficit on real and nominal union variables. Finally, section 5 concludes and of-

[1] Several recent works have made use of this framework, including Benigno 2000 and Woodford 1996.

fers directions for future work.

2 The model of a two-country monetary union

The model used to represent a monetary union with two countries is a standard general equilibrium model with monopolistic competition and nominal price rigidity á la Calvo. The union is populated by a continuum of households who are both consumers and producers of differentiated goods and live in two different countries: country 1 and country 2. They are indexed by $j \in [0,1]$. Let households $j \in J_1 = [0,n)$ belong to country 1 and households $j \in J_2 = [n,1]$ belong to country 2. Households are assumed to have the same preferences within and across countries in the union. Each household is taxed by and enjoys public goods offered by the government of the country she belongs to. The government of each country $i = 1,2$ levies taxes (lump-sum taxes in the simplified version of the model and distorting, proportional tax on nominal income in the general case) upon each household in J_i, and produces public goods in quantity G_t^i in period t. These goods benefit only the households in J_i. Prices are rigid, in each period only a share $1-\alpha$ of the population, chosen randomly, is allowed to change the price of the good it supplies. As a result, the level and pattern of economic activity is distorted by price-level variations over time. The monetary policy of the union is managed by a common central bank which is assumed to follow a monetary policy totally independent of the fiscal policy of the two governments.

We only consider equilibria in which all state variables follow paths that are close to the values they would have in a stationary deterministic equilibrium. In order to characterize equilibria of this kind, we consider a log-linearization of the system of equilibrium conditions around the steady state.

A detailed description of the model is redundant and is relegated in the appendix. Here, I only describe the equilibrium conditions, divided into an aggregate demand block and an aggregate supply block and their log-linearized version (around a deterministic steady state).

2.1 The simplified model with one good and lump sum taxes

In the union, households receive utility out of consumption of private and public goods and disutility out of producing one good. Households demand without distinction goods produced at home and abroad (such goods enter into the utility function as perfect substitutes). Two distinct governments offer pubic goods to their own citizens. They also demand both domestic and foreign goods. Households are also producers of differentiated goods in a monopolistically competitive environment in which prices are not flexible. There is a monetary authority in the union that decides the monetary policy of the union. The union can be modelled as a closed economy with the difference that here the government budget constraint combines the budget constraints of the two governments of the union[2]. Indeed, it is the aggregate level of outstanding debt that matters for price determination.

2.1.1 Aggregate demand block

The aggregate demand block of the model reduces to the following system of equations

$$\beta \frac{u'(Y_{T+1})}{u'(Y_T)} \frac{P_T}{P_{T+1}} = (1+i_T)^{-1}, \tag{1}$$

$$i_T = \Theta(\pi_T, Y_T, \xi_T), \ where\ \xi_T = \rho_1 \xi_{T-1} + \varepsilon_{1T}, \\ \mid \rho_1 \mid \leq 1,\ \varepsilon_{1T} \sim N(0, \sigma_1^2), \tag{2}$$

$$\Delta_T = \Gamma\left(\frac{B_T}{P_{T-1}}, \zeta_T\right), \ where\ \zeta_T = \rho_2 \zeta_{T-1} + \varepsilon_{2T}, \\ \mid \rho_2 \mid \leq 1,\ \varepsilon_{2T} \sim N(0, \sigma_2^2), \tag{3}$$

$$B_{T+1} = (1+i_T)\left[B_T + P_T \Delta_T\right], \tag{4}$$

[2]See appendix for details.

where Y_T, Δ_T are union-wide output and budget deficit respectively; B_T is the compounded debt of the two governments accumulated at the end of period $T-1$; and i_T is the common nominal interest rate on a riskless one-period nominal bond issued by the individual governments and purchased at T. Equation (2.1) is the usual consumption Euler equation. Here output and total consumption coincide. Notice that, since there are complete contingent markets, households have identical preferences, and we assume that they have identical initial wealth, then they choose to pool the risk associated with price setting and choose identical total consumption plans - where total consumption is here equal to private plus public consumption. Equation (2.2) describes the monetary policy rule followed by the common central bank. The central bank adjusts the short-term nominal interest rate in response only to inflation, output gap. Thus, the monetary authority acts independently of the two fiscal authorities. Equation (2.3) combines the rules adopted by the individual fiscal authorities. The union is subjected to two policy shocks, ε_{1T}, ε_{2T}, which are serially and mutually uncorrelated, i.e., $E\varepsilon_{iT}\varepsilon_{jT-k} = 0$ for all k and $i, j = 1, 2,\ i \neq j$. The rules consist of systematic policy responses to economic conditions (the responses to π_T, Y_T and $\frac{B_T}{P_T}$) and to random shocks (ξ_T and ζ_T). An authority can adopt an active policy when it is not constrained by budgetary conditions[3]. Here, the monetary authority is active when it reacts to a one percent increase in inflation by raising the nominal interest rates by more than one percent, and it acts passively when it obeys the constraints imposed by private behavior and fiscal policy and allows the money stock to respond to deficit shocks. The fiscal authority responds actively when it refuses to adjust direct taxes, preventing deficit shocks from being financed by future taxes. It acts passively when it obeys the constraints imposed by private and monetary policy behavior and passively adjusts direct taxes to balance the budget. Notice that the aggregate intertemporal budget constraint, summarized by $\lim_{T\to\infty} E_t\left[R_{t,T}B_T\right] = 0$, can be ignored as we are considering only solutions in which interest rates, inflation and real financial wealth remain forever near the stationary equilibrium path, in any of which this condition is satisfied[4]. The last equation, (2.4), represents the budget constraint of the two governments - which are identical in equilibrium in this exercise..

The system can be solved for the paths of $\{Y_T, i_T, B_{T+1}, \Delta_T\}_{T=t}^{\infty}$ given $\{P_T\}_{T=t}^{\infty}$, the exogenous shocks ε_1 and ε_2, and the initial conditions B_t, P_{t-1}.

2.1.2 Aggregate supply block

The aggregate supply block of the model reduces to the following system of equations

$$\sum_{k=0}^{\infty} \alpha^k E_t \left\{ R_{t,t+k} Y_{t+k} \left(\frac{P_t^*}{P_{t+k}}\right)^{-\theta} \left[P_t^* - \mu S_{t+k,t}\right] \right\} = 0, \tag{5}$$

$$S_{T,t} = \frac{w'\left(Y_T\left(\frac{P_t^*}{P_{t+k}}\right)^{-\theta}\right)}{u'\left(Y_T\right)} P_T, \tag{6}$$

[3] The fiscal authority and monetary authority can either accommodate itself to the policy of the other authority or not do so, thus being "passive" or "active" in Leeper's terms. Specifically, the terms active and passive policy refer to the constraints a policy authority faces. An active authority sets its control variable freely as to maximize its objective function. A passive authority, on the other hand, in responding to such shocks, is constrained by the behavior of private agents and by the action of the active authority. Thus, four combinations of fiscal and monetary policy are possible.

[4] We abstract from the liquidity services provided by money. This is no more than a simplification. Our model can be understood as the limit of a model where real money balances provide utility but where, in the limit, these liquidity services are arbitrarily small. Alternatively, the model can be understood as one where utility is additively separable in real money balances, consumption, and goods supply, as in Woodford (1996). In this case the model implies an additional first order condition relating real money balances to consumption and the interest rate. In the presence of any interest rule, the additional equilibrium condition simply determines the nominal level of money balances. Since this equilibrium condition plays no role in determining inflation, output or interest rates, it can safely be ignored for our purposes. Woodford (1998).

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha) P_t^{*(1-\theta)}\right]^{\frac{1}{1-\theta}}, \quad (7)$$

where $\mu \equiv \frac{\theta}{\theta-1} > 1$. Equation (2.5) is the first order condition for maximization of the current value of the future stream of profits of the household/producer of a differentiated good and P_t^* is the maximizing price. Equation (2.6) describes the evolution of the marginal cost of production at T of a good whose price was changed at t and is denoted by $S_{T,t}$. The last equation describes the evolution of the price index. The system can be solved for the paths of $\{P_T, P_T^*\}_{T=t}^{\infty}$ given $\{Y_T\}_{T=t}^{\infty}$ and the initial condition P_{t-1}.

The aggregate demand and supply blocks constitute a set of conditions that must be satisfied by a rational expectation equilibrium, i.e., a sequence of $\{P_T, P_T^*, Y_T, i_T, \Delta_T, B_T\}_{T=t}^{\infty}$ given the exogenous sequence $\{\varepsilon_{1T}, \varepsilon_{2T}\}_{T=t}^{\infty}$ of shocks affecting the economy.

2.1.3 The Log-linearized Model

The log-linearized aggregate demand block is given by

$$\hat{Y}_T = E_T \hat{Y}_{T+1} - \sigma\left(\hat{i}_T - E_T \hat{\pi}_{T+1}\right), \quad (8)$$

$$\hat{i}_T = \phi_\pi \hat{\pi}_T + \phi_Y \hat{Y}_T + \varepsilon_{1T}, \quad (9)$$

$$\hat{\Delta}_T = -\gamma \hat{b}_T + \varepsilon_{2T}, \quad (10)$$

$$\hat{b}_{T+1} = \hat{i}_T + \beta^{-1}\left(\hat{b}_T - \hat{\pi}_T\right) + \left(\beta^{-1} - 1\right)\hat{\Delta}_T, \quad (11)$$

where $\hat{Y}_T, \hat{\pi}_T, \hat{b}_T$ denote percentage deviations of $Y_T, \pi_T, b_T = \frac{B_T}{P_{T-1}}$ respectively from their stationary values, and $\hat{\Delta}_T \equiv \frac{\Delta^* - \Delta}{\Delta^*}$; the coefficient $\sigma = -\frac{u'(Y^*)}{u''(Y^*)Y^*}$, which is the elasticity of substitution between consumption at different dates.

The log-linearized aggregate supply block is given by[5]:

$$\hat{\pi}_T = \beta E_T \hat{\pi}_{T+1} + \kappa \hat{Y}_T, \quad (12)$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{w+\sigma}{\sigma(w+\theta)}$ and $w \equiv \frac{w'(Y^*)}{w''(Y^*)Y^*}$, which is what the elasticity of supply of a price-taking household would be to a change in the price at which it could sell one of the goods that it produces. This equation determines the equilibrium path of inflation associated with a given $\left\{\hat{Y}_T\right\}_{T=t}^{\infty}$ and represents an expectation-augmented Phillips curve.

2.2 The issue of determinacy

Equation (2.9) (2.10) can be substituted into the system to get the following linearized system, whose properties can be analyzed by shocking the exogenous variables, i.e., ε_{1T} and ε_{2T}.

$$E_T \hat{X}_{T+1} = M \hat{X}_T + R\Sigma_T, \quad (13)$$

$$M = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ \sigma\left(\phi_\pi - \frac{1}{\beta}\right) & 1 + \sigma\phi_y + \sigma\frac{\kappa}{\beta} & 0 \\ \phi_\pi - \frac{1}{\beta} & \phi_y & \frac{1}{\beta} - \gamma\left(\beta^{-1} - 1\right) \end{bmatrix} \quad (14)$$

$$\hat{X}_T' = \left[\hat{\pi}_T, \hat{Y}_T, \hat{b}_T\right], \quad \Sigma_T = \begin{bmatrix} \varepsilon_{1T} \\ \varepsilon_{2T} \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 \\ \sigma & 0 \\ 1 & \beta^{-1} - 1 \end{bmatrix}, \quad (15)$$

[5] Obtained by log-linearizing the aggregate supply block and substituting to eliminate P^*.

In the present case in which there is one predetermined variables, a sufficient condition for a unique saddle-path equilibrium is that two roots of M lie inside the unit circle and one lie outside (Blanchard-Khan 1980). It is easy to show that this happens when $\phi_\pi, \gamma > 1$ and when $\phi_\pi, \gamma < 1$, i.e., when one authority's behavior is active and that of the other is passive. Active behavior completely specifies policy and uniquely determines the equilibrium pricing function. Passive policy prevents an explosive path of government debt. If both authorities behave passively, then there are many processes for the level of prices that are consistent with equilibrium. This reproduces Sargent and Wallace's (1975) price-level-indeterminacy result. If both authorities are active, then there does not exist a process for the level of prices that ensures that consumers will hold government debt unless the policy shocks are related in a way that violates the assumption of mutually uncorrelated shocks. This can be seen clearly from the aggregate government budget constraint (2.4) that depends on both the interest rate, under the control of the monetary authority, and aggregate deficit, under the control of fiscal authorities. Thus, uncorrelated policies that arbitrarily vary both money and deficit violate the budget constraint.

3 Fiscal policy under active monetary policy

A combination of active and passive policies compatible with a unique and bounded equilibrium entails the central bank credibly committed to responding to inflation raising the nominal interest rate by more than the increase in inflation, i.e., in terms of the model, that $\phi_\pi > 1$.Under this monetary policy rule, monetary policy uniquely determines prices and quantities within the union. This can be seen by solving the system composed by the Euler equation (2.1), the monetary policy rule (2.2) and the aggregate supply (2.5). This system has a unique and bounded solution for $\phi_\pi > 1$. Once monetary policy has determined equilibrium prices and quantities within the union, each individual fiscal authority can either comply, and opt for a Ricardian fiscal policy, and remain in the union, or it can decide not to. Under these circumstances an equilibrium does not exists. As illustrated in section 2.2, with a pair of active policies there does not exist a process for the level of prices that ensure that consumers will hold government debt. This situation would result in default of the country or in its abandonment of the union. In both cases, the fiscal shock would be absorbed within the country without spreading disturbances to the rest of the union, in particular, without affecting the level of prices.

Thus, under its mandate, the European Central Bank should credibly 'threaten' to raise the interest rate sufficiently in response to any surge in inflation. Such a threat would not have to be carried out in practice, since its very existence would make the equilibrium with zero inflation unique[6].

The result is so far in line with Chari and Kehoe 1998, in which they prove that when the monetary authority cannot commit to a rule, the result, in terms of welfare, of a non-cooperative behavior of fiscal authorities is lower that with cooperation. On the other hand, when the central bank can credibly commit, the two results coincide.

3.1 Robustness of the determinacy result

This section enriches the model of a monetary union previously described to test the robustness of the result we have obtained so far. We maintain the same kind of price rigidity and the same structure of preferences across individuals in the union. But, we assume that domestic and foreign goods do not enter in the utility function as perfect substitute as before, but with an elasticity of substitution equal to one[7]. This is done in order to account for the very realistic possibility that individual countries face different domestic inflations in spite of facing a common level of prices[8].

[6] The European Central Bank is mandated to maintain price stability within the union and the art 104a of the Treaty 'bans direct central bank financing and access to favorable financing of public deficits'.

[7] See Obsfeld, Rogoff 1998 for a motivation of this convenient choice.

[8] See appendix for details.

We continue to assume complete asset markets. We examine the effects of an unexpected shock to government primary deficit. We assume that the central bank is committed to a rule such that the nominal interest rate responds to consumer price inflation with a coefficient greater than one. Such inflation is assumed to be weighted average of domestic inflations where the weights correspond to the size of the country. We check whether such policy leads to a unique equilibrium of prices and quantities in the union - as it was the case in the simplified model.

The system we need to solve in order to check whether monetary policy alone is able to determine uniquely prices and quantities of equilibrium is

$$\hat{Y}_T = E_T \hat{Y}_{T+1} - \sigma \left(\hat{i}_T - E_T \hat{\pi}_{T+1} \right), \tag{16}$$

$$\hat{i}_T = \phi_\pi \hat{\pi}_T + \phi_y \hat{Y}_T + \varepsilon_{1T}, \tag{17}$$

$$\hat{\pi}_T = \beta E_T \hat{\pi}_{T+1} + \kappa \hat{Y}_T, \tag{18}$$

where a variable $\hat{X}$ still denotes percentage deviations of X from its stationary value. Equation (3.1) is the Euler equation obtained as in (2.8), equation (3.2) is the usual rule of monetary policy and equation (3.3) is the union aggregate supply. It is, as before, the weighted average of the aggregate supplies of the two countries. Specifically, the aggregate supply in country 1 and 2 are respectively

$$\hat{\pi}^1_T = \beta E_T \hat{\pi}^1_{T+1} + \kappa_c \hat{C}^*_T + \kappa_Q \hat{Q}_T, \tag{19}$$

$$\hat{\pi}^2_T = \beta E_T \hat{\pi}^2_{T+1} + \kappa_c \hat{C}^*_T + \kappa_Q \hat{Q}_T, \tag{20}$$

where with $C^* = C+G$ we indicate total consumption and $Q \equiv P_2/P_1$ denotes the terms of trade[9]. The union inflation is then

$$\hat{\pi}_T = n \hat{\pi}^1_T + (1-n) \hat{\pi}^2_T. \tag{21}$$

Notice that the above system is identical to the one used in the simplified model and is again the system of a closed economy. Therefore, an active monetary policy, i.e., $\phi_\pi > 1$, uniquely determines union prices and quantities, in particular, the union inflation rate. The monetary authority is again able to maintain price stability within the union. The analysis continues with the determination of the inflation rate at the country level.

From the AS equations in country 1 and 2 we can derive the relative inflation as

$$\hat{\pi}^R_T = \hat{\pi}^2_T - \hat{\pi}^1_T = -\kappa_Q \hat{Q}_T + \beta E_T \hat{\pi}^R_{T+1}, \tag{22}$$

and noting that $\hat{\pi}^R_T = \hat{Q}_T - \hat{Q}_{T-1}$, we obtain a second order stochastic difference equation in $\hat{Q}$

$$E_T \hat{Q}_{T+1} - \frac{1+\beta+\kappa_Q}{\beta} \hat{Q}_T + \frac{1}{\beta} \hat{Q}_{T-1} = 0 \tag{23}$$

that has a unique, stable solution[10]. Furthermore, by inspection of (3.8), it appears that the terms of trade are completely insulated from monetary policy.

The latter results give us determinacy of the equilibrium also in this more complex model when monetary policy is active. Indeed, an active monetary policy uniquely determines union total consumption and prices. The law of motion of the terms of trade gives us relative inflation and then domestic inflations in the two countries. However, in an equilibrium with zero union inflation temporary domestic inflations different from zero are possible.

A further experiment could be conducted in the case of different degrees of nominal rigidity between the two countries, that is, $\alpha^1 \neq \alpha^2$.

The system to be solved to analyze the issue of determinacy of the equilibrium in this case would become

[9] See Benigno 2000 for the derivation of the AS curve. The coefficients $\kappa_C \equiv [(1-\alpha\beta)(1-\alpha)/\alpha] \cdot [(\sigma+\omega)/(1+\theta\omega)]$ and $\kappa_Q \equiv \kappa_C \cdot [(1+\omega)/(\sigma+\omega)]$, where σ and ω are as already defined. Here $\pi^1_T = \ln P_{1,T}/P_{1,T-1}$ and $\pi^2_T = \ln P_{2,T}/P_{2,T-1}$ and $P_{1,T}$, $P_{2,T}$ are the prices of the bundles of goods produced respectively in country 1 and 2.

[10] See Benigno 2000.

$$\hat{Y}_T = E_T\hat{Y}_{T+1} - \rho\left(\hat{i}_T - E_T\hat{\pi}_{T+1}\right)^{11}, \qquad (24)$$

$$\hat{i}_T = \phi_\pi\hat{\pi}_T + \phi_y\hat{Y}_T + \varepsilon_{1T}, \qquad (25)$$

$$\hat{\pi}_T = \beta E_T\hat{\pi}_{T+1} + \kappa\hat{Y}_T + \rho\hat{Q}_T, \qquad (26)$$

where, clearly, relative prices are important and the trade-off between stabilizing union inflation and union output may emerge[12].

The relative inflation in this case becomes

$$\hat{\pi}_T^R = \hat{\pi}_T^2 - \hat{\pi}_T^1 = -\psi\hat{Q}_T + \omega\hat{Y}_T + \beta E_T\hat{\pi}_{T+1}^R. \quad (27)$$

Notice that here monetary policy has an influence on the terms of trade, through the influence on union total consumption or union output. Equation (3.12) can be rewritten as

$$E_TQ_{T+1} = -\omega\hat{Y}_T - \frac{1}{\beta}(1+\psi+\beta)Q_T - Z_T, \quad (28)$$

where Z_T has been defined as $Z_T \equiv Q_{T-1}$. The resulting system of four equations becomes

$$E_T\hat{X}_{T+1} = M\hat{X}_T, \qquad (29)$$

$$\hat{X}'_T = \left[\hat{\pi}_T, \hat{Y}_T, \hat{Q}_T, \hat{Z}_T\right],$$

$$M = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & -\frac{\sigma}{\beta} & 0 \\ \sigma\left(\phi_\pi - \frac{1}{\beta}\right) & 1+\sigma\phi_y+\sigma\frac{\kappa}{\beta} & -\frac{\sigma^2}{\beta} & 0 \\ 0 & -\omega & \frac{1+\psi+\beta}{\beta} & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (30)$$

[12] Differently from the previous two cases in which the union could be modelled as a closed economy. The absence of such trade off is a well known result of this kind of models of general equilibrium of closed economies.

Since there is one predetermined variable, it can be shown that again an active monetary police, i.e., $\phi_\pi > 1$, is sufficient to uniquely determine union quantities and prices[13].

To conclude, monetary policy suffice for price stability while again controls on fiscal policy are required in order for fiscal policy to be Ricardian and thus for an equilibrium to exist.

3.2 Is the central bank fully credible?

A common concern has to do with the credibility of the central bank in adopting an active monetary policy no matter what is the situation of the debt in the union members. The monetary authority may ex-post find it optimal to actually bail-out a country in financial distress. The Treaty is explicit in forbidding any monetization of the debt of some of the member countries. However, possible deviations can be envisaged. If such deviations are perceived by the financial markets, the credibility of the central bank would be undermined and with it its ability to achieve its foremost objective of maintaining price stability. This considerations would make the coordination of fiscal and monetary policy an imperative for the achievement of price stability.

4 Monetary policy when fiscal policy is active

When households expect offsetting adjustments to fiscal shocks -Ricardian equivalence- then public debt is not perceived as net wealth and shocks to the primary deficit do not generate wealth effects. A fiscal policy that displays this feature is called 'Ricardian'[14]. A fiscal policy has this property if it adjusts the size of the real primary budget deficit so as to prevent the real value of outstanding government debt from exploding, regardless of the paths of endogenous variables. Analytically, a fiscal policy is 'Ricardian' if the equilibrium condition $\lim_{T\to\infty} E_T[R_{t,T}B_T] = 0$

[13] The result contrasts with Beetsema and Jensen's who find indeterminacy in the case of symmetric shocks.

[14] The terminology used here is as in Woodford 1996.

holds as an identity. In order for fiscal policy to have this feature it is sufficient to impose constraints on its asymptotic behavior[15].

However, when considering the monetary union as a union of 'regions within a closed economy lubricated by a common currency' (Mundell 1961), Ricardian equivalence does not necessarily hold, because, acting as leader, the fiscal authorities do not necessarily opt for a Ricardian fiscal policy. In this case, the aggregate public debt of the union and the consolidated deficit matter for the determination of the equilibrium path of inflation, interest rates and output.

With active fiscal policy, possibly not Ricardian, the best response of the monetary authority to the behavior of agents and of the fiscal authority can be deduced by simulating the response of a two-country monetary union economy to a shock to the combined primary deficits of the two governments. I assume $\gamma = 0$, that is, the union deficit follows a stochastic process, $\hat{\Delta}_T = \varepsilon_{2T}$[16] This is clearly an example of active fiscal policy, as it is totally unconstrained and independent of agents' behavior and monetary policy. The simulation that follows will use the procedure suggested in Blanchard-Khan (1980). The numerical simulation seeks to analyze the impact of a positive shock to the compounded fiscal deficit of the union on nominal and real variables, in particular, real union-wide output, the common inflation rate, the common short-term nominal interest rate, the ex-ante common real rate $\hat{r}_T = \hat{i}_T - E_T \hat{\pi}_{T+1}$, the consolidated real government debt, to disturbances in the path of the consolidated real primary deficit.

The parameters of the model are assigned the values $\beta = 0.95$, implying a time preference of 5 percent per year, consistent with observed real rates of return; $\kappa = 0.3$, $\gamma = 0.1$ are consistent with econometric estimates for the US; $\sigma = 1$ are obtained by assuming logaritmic preferences for consumption, which is a standard assumption consistent with micro data. I initially consider a pure interest peg imposed by the common central bank, i.e., $\phi_\pi = 0$, then an aggressive monetary policy compatible with passive behavior, $\phi_\pi = 0.9$, which is near the upper bound for a passive monetary policy.

The responses to a shock to government deficit are consistent with those in Woodford (1996) for a closed economy. The unexpected increase in the consolidated primary deficit of at least one country if it is not offset by any expected reduction in its future primary deficits - stimulates aggregate demand in that country, and thus in the union, temporarily increasing both inflation and output. Ricardian equivalence clearly fails to hold in spite of the assumption of rational expectations, identical infinitely-lived households, pure lump-sum taxation, and frictionless financial markets, and despite the assumption that the monetary authority is completely independent of the fiscal authorities of the two countries. This happens because an increase in the present value of at least one of the two government deficits increases the present value of the total consumption that the representative household of that country can afford - at unchanged prices and interest rates - and thus induces an increase in the aggregate demand for goods at those given prices.

Indeed, so long as the value of government liabilities exceeds the present value of expected government budget surpluses, households will be able to afford total consumption with a present value greater than present value of the economy's output. Equilibrium requires adjustment of prices and/or of interest rates so as to preserve equality between the value of outstanding government liabilities and the present value of future government surpluses. In the context of a monetary union, these changes will affect all the member countries, independently of the way in which

[15] However, for fiscal policy to be actually neutral it has to be perceived as 'Ricardian' by households. A commitment from the part of the government to constrain its fiscal policy asymptotically might not be sufficiently credible, thus the commitment to constrain the path of government deficit and/or public debt in the near future as well. One can show that a ceiling over the real value of the public debt at all t, as stated in the Maastricht Treaty, suffices to guarantee that the limit holds as an identity. However Ricardian, such criteria are perceived as to be too arbitrary and rigid - a common concern is their inability to be countercyclical that may lead to unecessary deep recessions.

[16] This assumption, although unrealistic, serves the purpose. In addition, it can be shown that even assuming a certain degree of serial correlation in the deficit does not change the results qualitatively (Piffanelli 1999a).

they conduct fiscal policy.

In the present model, when aggregate demand increases, inflation also increases and real interest rates fall. In equilibrium, the actual increase in demand is just enough to produce a capital loss and a real interest rate decline that suffice to prevent households from being able to afford more goods than the economy supplies. Higher inflation is a direct result of a demand for goods above potential so that the marginal cost is higher than the general price index divided by the mark-up, μ. Prices set at the time of the shock are thus higher than the price index in the previous period, resulting in inflation. Given the monetary policy assumed here, nominal interest rates increase but, as $\phi_\pi < 1$, the ex-ante real interest rate declines. The increase in prices drives output back to equilibrium. However, an output temporary higher than potential requires real rates to decline so that households are induced to consume more in the period after the shock occurred.

Specifically, a 20 percent unexpected increase in the combined primary budget deficit at $t = 1$ results in an increase in inflation, in real GDP, and a slight rise in nominal interest rates that implies a decline in the real interest rate. The effect on combined public debt depends on how much real rates decline. The interesting feature of this specification is that a more aggressive monetary policy of the central bank would amplify and prolong the effects of the shock on the other variables.

Figure 1 shows the impulse responses of several variables to a 20 percent unexpected increase in the compounded primary government budget deficit when the monetary authority responds by pegging the interest rate, $\phi_\pi = 0$. Figure 2 shows the responses to the same shock when the monetary authority reacts aggressively, $\phi_\pi = 0.9$. From a comparison of figures 1 and 2 it appears that, in this setup, the best response of the central bank to a fiscal shock is a pure peg (fig 1). For example, with a peg, in response to a 20 percent shock in the compounded government deficit inflation rises by less than 0.5 percent, thus real interest rate falls by 25 basis points, real union GDP increases by more than 0.5 percent and the sum of the two real government debts increases by 0.5 percent on impact and returns back to the steady state in the next few periods. On the other hand, an aggressive response on the part of the central bank to excess capacity and especially to inflation, would worsen the situation in this model (fig 2). A 20 percent unexpected increase in the combined primary deficit is followed by a rise in inflation double than in the pure peg case, and as the real rate does not change significantly, the nominal rate increases by the magnitude of inflation. The Union's output increases slightly. The most unpleasant result is an increase of the compounded outstanding debt by more than 2 percent and all disturbances die out very slowly. A bit of paramenter sensitivity analysis is also performed. Figure 3 depicts the flexible prices case. The responses to the shock indicates that price sluggishness is crucial for the previously described results to hold. Real effects of fiscal shocks are still noticeable, but they tend to day out within a period. In figure 4 the seignorage[17] channel is completely shut down, i.e., $\nu = 0$. The figure shows that the qualitative results are unchanged[18].

4.1 Responses at the country level

Differently from the closed economy case, in a monetary union, even if one government is fiscally responsible, for example by keeping its real primary deficit at some sustainable constant level, variations in the budget deficit of the other government not backed by taxes will result in price level instability throughout the union. Consequently, there is a clear reason for a government concerned about joining a monetary union to care about the fiscal policies of the other governments with which it shares a common currency.

To see why this is happening, notice that in equilibrium the transversality condition $\lim_{T\to\infty} E_t \left[R_{t,T} B_T\right] = 0$ must hold, where $B_T \equiv \left\{\frac{B_T^1 + B_T^2}{2}\right\}$ is the financial wealth of the union.

However, the above equality does not need to be

[17] Seignorage represents the real revenues a government acquires by using newly issued money to buy goods and non-money assets (Obstfeld-Rogoff 1996)

[18] This piece of evidence indicates the importance of fiscal policy in price level determination besides the seignorage channel, often considered the main driving force.

true for the present value of each country's public debt. Indeed, consider the case in which country 1 keeps a primary deficit constant forever $\Delta^1 = \frac{\Delta^*}{2}$ consistent with stable prices in the sense that its current value equals the value of outstanding debt, but at t country 2 runs a primary deficit $\Delta_t^2 = \frac{\Delta^*}{2} + \varepsilon$ with $\varepsilon > 0$ and afterwards $\Delta_t^2 = \frac{\Delta^*}{2}$ at any $T \neq t$. In the case of an equilibrium where $\lim_{T\to\infty} E_t [R_{t,T} B_T] = 0$ holds, then it must be the case that

$$\begin{aligned} &\lim_{T\to\infty} E_t \left[R_{t,T} B_T^1\right] = P_t \tfrac{\varepsilon}{2}, \\ \text{and } &\lim_{T\to\infty} E_t \left[R_{t,T} B_T^2\right] = -P_t \tfrac{\varepsilon}{2}. \end{aligned} \tag{31}$$

This means that the outstanding debt of country 2 grows at the rate of interest forever while country 1 becomes a net creditor, with the amount of credit extended by that government growing at the rate of the interest rate as well. In effect, the government of country 1 lends to that of country 2, purchasing a quantity $P_t \frac{\varepsilon}{2}$ of the debt issued by government 2 at t, and rolling the loan over forever, never demanding repayment.

In addition, if country 1 decides not to collaborate, the cost it will have to pay will be price instability in the union. To see what happens, consider the case in which Ricardian equivalence holds in country 1 no matter what country 2 does. The effect of a shock in country 2 in the aggregate would be as if government 1 were to vary its own budget in perfect lock-step with that of country 2, so that the public debts of the two countries always grow at exactly the same rate. This can be seen from the union-wide budget constraint

$$\sum_{T=t}^{\infty} E_t \left\{R_{t,T} \left[P_T \Delta_T\right]\right\} = B_t \tag{32}$$

and from the fact that Ricardian equivalence in country 1 implies

$$\sum_{T=t}^{\infty} E_t \left\{R_{t,T} \left[P_T \Delta_T^1\right]\right\} = B_t^1. \tag{33}$$

Then, subtracting (4.2) from (4.1) and multiplying the result by 2, I can write

$$\sum_{T=t}^{\infty} E_t \left\{R_{t,T} \left[2 P_T \Delta_T^2\right]\right\} = 2B_t^2, \tag{34}$$

but this is exactly the form that the union-wide intertemporal budget constraint would take if the fiscal policy rule of government 1 were given by $\Delta_t^1 = \Delta_t^2$ at all times, so that $B_t^1 = B_t^2$ at all times as well. This would mean that whenever government 2 reduces the present value of its budget surplus, government 1 reduces its own as well, by the same amount, thus doubling the inflationary impact of the expansionary fiscal policy on the part of government 2.

Intuitively, if government 1 chooses not to cooperate by financing some of the budget deficit of government 2, then prices and interest rates would need to be adjusted even more in order to restore the equilibrium between private sector purchasing power and the quantity of output available for households to purchase.

To conclude, the only way in which government 1 can act to minimize the macroeconomic instability resulting from fiscal instability in country 2 is for it to adjust the size of its budget deficit inversely with that of government 2.

5 Conclusions

The paper attempted to explore whether, in the context of the European Monetary Union in which price stability is by constitution the main objective of the sole monetary authority, the fiscal criteria of the Maastricht Treaty for joining the union are necessary for achieving the above-mentioned objective. The analysis has been conducted using a standard general equilibrium model featuring monopolistic competition and nominal rigidities that describes a monetary union with two countries.

In the paper we have found a rationale for imposing constraints on individual fiscal policies within a monetary union. Using different settings we have concluded that when price stability is the sole objective of the monetary authority, then monetary policy

alone is able to reach such objective so long as it can credibly commit to a rule in which the nominal interest rate is raised sufficiently in response to any surge in inflation. Indeed, monetary policy alone determines a unique and bounded equilibrium of union quantities and prices. However, at such prices - and interest rate - the individual fiscal authorities have to cooperate by following a Ricardian fiscal policy. If they do not, then the government debt would not find a market and the countries experiencing fiscal imbalances would have to face the choice of either defaulting or abandoning the monetary union.

In addition, when the monetary authority cannot credibly commit itself to a rule, fiscal irresponsibility from the part of at least one government can be compatible with equilibrium, but monetary policy alone is no longer able to guarantee price stability. This happens because Ricardian equivalence does not necessarily hold at the country level in this model. Thus, fiscal irresponsibility of one of the two governments spreads disturbances across the union undermining any attempt by the sole monetary authority to maintain stable prices. Further, I have shown that the only way a fiscally responsible government can act to minimize the macroeconomic instability resulting from fiscal irresponsibility in the other country is for it to adjust the size of its budget deficit inversely with that of that government. Such concerns may have motivated the architects of the Maastricht Treaty in the direction of fiscal rigorousness.

An extension of this paper would try to envisage combinations of active monetary policies and passive fiscal policies leading to determinate and bounded equilibria compatible with price stability within the union. One such combination has already been identified in an active monetary policy and fiscal policies as indicated in the Treaty. Alternative combinations that impose less constraints on fiscal authorities and be welfare improving can be found.

The existing literature on optimal monetary policy[19] concludes that price stability is indeed the optimal policy for a central bank that aim at maximizing its objective function when such objective is the economy utility-based welfare. However, such result is obtained under the assumption that the role of fiscal policy is merely to neutralize the distorting effect of monopoly power so that monetary policy is left with the task of neutralizing the distortions deriving from price rigidity. The rationale is that welfare is maximized when the economy operates in an efficient way and thus policies can be used to achieve such efficiency. Moreover, price stabilization may no longer be the optimal monetary policy when the economy operates inefficiently, that is when fiscal policy does not offset monopolistic distortions. The search of a combination of optimal fiscal and monetary policies mutually compatible with price stability could be the object of future research.

[19] For example Rotemberg, Woodford (1998) and Woodford (1999).

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6 APPENDIX:

The 2-country monetary union can be represented by a standard general equilibrium model with monopolistic competition and price rigidity. The union is populated by a continuum of households who are both consumers and producers of differentiated goods and who live in the two countries: country 1 and country 2. They are indexed by $j \in [0,1]$. Let households $j \in J_1 = [0,n)$ belong to country 1 and households $j \in J_2 = [n,1]$ belong to country 2. Each household is taxed by and enjoys public goods offered by the government of the country she belongs to. The government of each country $i = 1,2$ levies taxes upon each household in J_i, and produces public goods in the quantity G_t^i in period t, which goods benefits only the households in J_i. Price rigidity follows a Calvo formulation: in each country and in each period a share $1-\alpha$ of the population chosen randomly, is allowed to change the price of the good it supplies. As a result of this price rigidity, the level and pattern of economic activity is distorted by price level variations over time. The only kind of uncertainty present in the model comes from this system of prices that does not allow to set prices at all dates. We assume a complete market of contingent claims, both domestically and internationally, such to span all the states of nature. That is, households are completely insured against their idiosyncratic income risk.

As a consumer, each household j, independently from the country she lives, seeks to maximize a lifetime objective

$$E_t\left\{\sum_{T=t}^{\infty}\beta^T\left[u\left(C_T^j+G_T^i\right)-w\left(y_T\left(j\right)\right)\right]\right\}, \quad j\in J^i,\ i=1,2 \tag{A.1}$$

where u is an increasing concave function, w is an increasing convex function, and β is the discount factor, with $0<\beta<1$. $y_T(j)$ is the household's supply of the good she produces and C^j is an index of consumption defined as

$$C^j \equiv \frac{\left(C_1^j\right)^n\cdot\left(C_2^j\right)^{1-n}}{n^n\left(1-n\right)^{1-n}}, \tag{A.2}$$

where C_1^j and C_2^j are indexes of consumption across the continuum of differentiated goods produced in country 1 and 2, and defined as.

$$C_1^j \equiv \left[\left(\frac{1}{n}\right)\int_0^n c_1^j(z)^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}}, \qquad C_2^j \equiv \left[\left(\frac{1}{1-n}\right)\int_n^1 c_2^j(z)^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}}, \tag{A.3}$$

where $c^j(z)$ indicates household's j consumption of good z in period T, and $\theta > 1$ is the constant elasticity of substitution among alternative goods.

The consumer price index in country i is defined as

$$P^i \equiv \left(P_1^i\right)^n \cdot \left(P_2^i\right)^{1-n}, \tag{A.4}$$

and the price indexes of goods produced in the two countries are

$$P_1^i \equiv \left[\int_0^n p_T^i(z)^{1-\theta}\,dz\right]^{\frac{1}{1-\theta}}, \qquad P_2^i \equiv \left[\int_n^1 p_T^i(z)^{1-\theta}\,dz\right]^{\frac{1}{1-\theta}}, \tag{A.5}$$

where $p^i(z)$ is the price in country i of good z. The above price indices are those that minimize the expenditures required to buy C^j, C_1^j and C_2^j respectively.

Similar reasoning holds for public consumption, that enters in the utility function as perfect substitute for private consumption. In particular,

$$G^i \equiv \frac{\left(G_1^i\right)^n \cdot \left(G_2^i\right)^{1-n}}{n^n\left(1-n\right)^{1-n}}, \tag{A.6}$$

where G^1 and C^2 are indexes of consumption across the continuum of differentiated goods produced in country 1 and 2.

$$G_1^i \equiv \left[\left(\frac{1}{n}\right)\int_0^n g_1^i(z)^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}}, \qquad G_2^i \equiv \left[\left(\frac{1}{1-n}\right)\int_n^1 g_2^i(z)^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}}, \tag{A.7}$$

At any T, household j's dynamic budget constraint becomes

$$\int_0^1 p_T(z)\,c_T^j(z)\,dz + E_T\left[R_{T,T+1}B_{T+1}^j\right] \leq B_T^j + \left(1-\tau^i\right)p_T(j)\,y_T(j),\ j \in J^i,\ i = 1,2 \tag{A.8}$$

where B_{T+1}^j is the nominal value at $T+1$ of the bond portfolio the household holds at the end of period T, $R_{T,T+1}$ is the stochastic discount factor, and τ^i is a regional proportional tax on nominal income[20]. Adding the proper no-Ponzi game condition, I can write household j's intertemporal budget constraint as

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[\int_0^1 p_T(z)\,c_T^j(z)\,dz\right]\right\} \leq \sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[\left(1-\tau^i\right)p_T(j)\,y_T(j)\right]\right\} + B_t^j,\ j \in J^i,\ i = 1,2 \tag{A.9}$$

looking forward from each date t. Notice that money does not appear in either the budget constraint and the utility function. Monetary policy is specified in terms of an interest rule. Thus, the explicit introduction of money is not necessary[21].

[20]Notice that for notational simplicity, we have not explicity included markets in private claims, so all borrowing and lending is between consumers and the government. Since all consumers are identical, such claims will not ne traded in equilibrium; hence their absence will not affect the equilibrium.

[21]Perhaps the most surprising element of our preference specification, given that we are interested in monetary issues, is that we abstract from the liquidity services provided by money. This is no more than a simplification. Our model can be understood as the limit of a model where real money balances provide utility but where, in the limit, these liquidity services are arbitrarily small. Alternatively, the model can be understood as one where utility is additively separable in real money balances, consumption, and goods supply, as in Woodford (1996). In this case the model implies an additional first order condition relating real money balances to consumption and the interest rate. In the presence of nay interest rule, the additional equilibrium condition simply determines the nominal level of money balances. Since this equilibrium condition plays no role in determining inflation, output or interest rates, it can safely be ignored for our purposes. Woodford (1998).

As a supplier of goods, the household seeks to maximize her profit. If at T the household is allowed to set the price for her good, then she chooses p to maximize the current value of the stream of future profits

$$\sum_{k=0}^{\infty} \alpha^k \{\Lambda_T E_T [R_{T,T+k} p y_{T+k}(p)] - \beta^k E_T [w(y_{T+k}(p))]\} \tag{A.10}$$

given the demand faced by each producer and where Λ_T is the marginal utility for the household of additional money income at T.

The government of each country is assumed to issue only riskless one-period nominal debt denominated in the common currency. Let B_T^i denote the nominal value of country i public debt at the beginning of period T B_T^ievolves according to

$$B_{T+1}^i = (1+i_T)\left[B_T^i + P_T \Delta_T^i\right], \; i = 1,2 \tag{A.11}$$

where i_T is the common nominal interest rate and $\Delta_T^i \equiv G_T^i - \frac{1}{P_T}\tau^i \int_{j\in i} p_T(j)\, y_T(j)\, dj$ is the real primary deficit of country i in T.

The common central bank supplies whichever quantity of money households demand and sets the monetary policy according to a rule, here a Taylor rule with which it adjusts the short-term nominal interest rate in response to inflation and excessive output, $i_T = \Phi(\pi_T, Y_T)$.

This is the setup of the model.

7 Case one: one good, lump sum taxes

Here we simplify the above model to consider only one good and allow only for lump sum taxes[22]. The consumption index C_T^j reduces to

$$C_T^j \equiv \left[\int_0^1 c_T^j(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}. \tag{A.12}$$

[22] In order to eliminate the effects of fiscal policy changes other than those that result from changes in budgets.

Similarly,

$$G_T^i \equiv \left[\int_0^1 g_T^i(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \; i = 1,2, \tag{A.13}$$

and the price index P_T simplifies to

$$P_T \equiv \left[\int_0^1 p_T(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}, \tag{A.14}$$

where $p_T(z)$ is the price of good z at T. Again, the above price index is the one that minimizes the expenditures required to buy C_T^j and G_T^i.

Households across the union have the same preferences, face the same prices, rates of return and face the same demand for the goods they produce. Assume households' initial wealth is such to allow them identical budget sets, and they begin with the same distribution of existing prices for the goods they supply, then households choose the same level of total consumption, $C_T^1 + G_T^1 = C_T^2 + G_T^2$, and the same price for their goods, $P_T^*(j) = P_T^*$ for all j .

As usual, the household's problem can be solved in two steps. The first, or expenditure minimization by the household and by the government respectively, implies $c_T^j(z) = C_T^j \left(\frac{p_T(z)}{P_T}\right)^{-\theta}$ and $g_T^j(z) = G_T^i \left(\frac{p_T(z)}{P_T}\right)^{-\theta}$, and thus the demand for good z is $y_T(z) = Y_T \left(\frac{p_T(z)}{P_T}\right)^{-\theta}$ where $Y_T = C_T + G_T$ and $C_T = \int_0^1 C_T^j dj$, $G_T = G_T^1 + G_T^2$.

The relevant necessary and sufficient condition for an optimal household's consumption plan resulting from the solution of the second stage problem is

$$\beta^{T-t} \frac{u'(Y_T)}{u'(Y_t)} \frac{P_t}{P_T} = R_{t,T}, \tag{A.15}$$

and in order to rule out arbitrage opportunities $1 + i_t \equiv [E_t\{R_{t,t+1}\}]^{-1}$ then

$$\beta\frac{u'(Y_{t+1})}{u'(Y_t)}\frac{P_t}{P_{t+1}} = (1+i_t)^{-1}, \tag{A.16}$$

Where equation (A.15) is the usual consumption Euler equation.

Household's j intertemporal budget constraint with lump sum taxes, T, reduces to

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[\int_0^1 p_T(z)\, c_T^j(z)\, dz\right]\right\} \leq \sum_{T=t}^{\infty} E_t\left\{R_{t,T}\cdot\left[p_T(j)\, y_T(j) - T_T^i\right]\right\} + B_t^j, \quad j \in J^i,\ i = 1,2, \tag{A.17}$$

and in symmetric equilibrium it can be written as

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[P_T C_T\right]\right\} = \sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[P_T Y_T - T_T^i\right]\right\} + B_t^i,\ \ j \in J^i,\ i = 1,2 \tag{A.18}$$

Summing over households one can obtain the union-wide intertemporal budget constraint

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[P_T \Delta_T\right]\right\} = B_t \tag{A.19}$$

Where $\Delta_T \equiv \Delta_T^1 + \Delta_T^2$ is the consolidated primary deficit of the union and $B_t = B_t^1 + B_t^2$ is the aggregate public debt.

Moving to the supply side of the model, the P_t^* that maximizes the current value of the future stream of profits of the household/producer of a differentiated good satisfies the first order condition

$$\sum_{k=0}^{\infty} \alpha^k E_t\left\{R_{t,t+k} Y_{t+k}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\theta}\left[P_t^* - \mu S_{t+k,t}\right]\right\} = 0, \tag{A.20}$$

where $\mu \equiv \frac{\theta}{\theta-1} > 1$, and $S_{T,t}$ is the marginal cost of production at T of a good whose price was changed at t and is equal to

$$S_{T,t} = \frac{w'\left(Y_T\left(\frac{P_t^*}{P_{t+k}}\right)^{-\theta}\right)}{u'(Y_T)} P_T, \tag{A.21}$$

the calvo formulation implies that the price index evolves according to

$$P_T = \left[\alpha P_{T-1}^{1-\theta} + (1-\alpha) P_T^{*(1-\theta)}\right]^{\frac{1}{1-\theta}}, \tag{A.22}$$

The above is the set of conditions that must be satisfied by a rational expectation equilibrium, i.e. a sequence of $\{P_T, P_T^*, Y_T, i_T, B_T\}_{T=t}^{\infty}$ given the exogenous sequence $\{\Delta_T\}_{T=t}^{\infty}$ of consolidated deficits of the two governments.

In this setup, in the absence of any constraint on fiscal policy, one can show that stochastic variations in the compounded budget of the two governments interfere both with price stability and with macroeconomic stability in the union. Furthermore this happens regardless of the form of the monetary policy, including the one proposed here where the central bank acts independently from the two fiscal authorities[23].

8 Case two: general case

In the general case, assuming no transaction costs in transporting goods across regions, it follows that $p^1(1) = p^2(1)$ and $p^1(2) = p^2(2)$. This assumption coupled with the structure of preferences implies purchasing power parity, i.e., $P^1 = P^2$. However, since there are two additional prices, those of the two bundles of goods produced in the two countries, we need to define the terms of trade for country 2 as $Q \equiv \frac{P_2}{P_1}$.

The household's problem can be solved in two steps. The first, or expenditure minimization by the household and by the government respectively, implies the following demand functions for private consumption

[23] See Woodford (1996) for a formal proof.

$$c^j(1) = \left(\frac{p(1)}{P_1}\right)^{-\theta} Q^{1-n} C^j,$$
$$c^j(2) = \left(\frac{p(2)}{P_2}\right)^{-\theta} Q^{-n} C^j, \qquad \text{(A.23)}$$

and the following demand functions for public consumption

$$g^i(1) = \left(\frac{p(1)}{P_1}\right)^{-\theta} Q^{1-n} G^i,$$
$$g^2(2) = \left(\frac{p(2)}{P_2}\right)^{-\theta} Q^{-n} G^i. \qquad \text{(A.24)}$$

Therefore, the total demand of a good produced in country 1 and 2 respectively can be written as

$$y^d(1) = \left(\frac{p(1)}{P_1}\right)^{-\theta} Q^{1-n}\left(C + G^i\right),$$
$$g^2(2) = \left(\frac{p(2)}{P_2}\right)^{-\theta} Q^{-n}\left(C + G^i\right), \qquad \text{(A.25)}$$

where $C \equiv \int_0^1 C^j dj$.

The relevant necessary and sufficient condition for an optimal household's consumption plan resulting from the solution of the second stage problem is

$$\beta^{T-t}\frac{u'\left(C_T^i\right)}{u'\left(C_t^i\right)}\frac{P_t}{P_T} = R_{t,T}, \qquad \text{(A.26)}$$

and in order to rule out arbitrage opportunities $1 + i_t \equiv \left[E_t\left\{R_{t,t+1}\right\}\right]^{-1}$
then

$$\beta E_t\left[\frac{u'\left(C_{t+1}^i + G_{t+1}^i\right)}{u'\left(C_t^i + G_t^i\right)}\frac{P_t}{P_{t+1}}\right] = (1 + i_t)^{-1}, \quad \text{(A.27)}$$

where equation (A.26) is the usual consumption Euler equation.

In order to rule out arbitrage opportunities across countries

$$E_t\left[\frac{u'\left(C_{t+1}^1 + G_{t+1}^1\right)}{u'\left(C_t^1 + G_t^1\right)}\frac{P_t}{P_{t+1}}\right] = E_t\left[\frac{u'\left(C_{t+1}^2 + G_{t+1}^2\right)}{u'\left(C_t^2 + G_t^2\right)}\frac{P_t}{P_{t+1}}\right]. \qquad \text{(A.28)}$$

Perfect risk sharing in (total) consumption within the union requires $C^1 + G^1 = C^2 + G^2 = C + G$ at any time and in any state.

Aggregate demand in both countries becomes

$$Y^1 \equiv \left[\left(\frac{1}{n}\right)\int_0^n y^d(1)^{\frac{\theta-1}{\theta}} d1\right]^{\frac{\theta}{\theta-1}},$$
$$Y^2 \equiv \left[\left(\frac{1}{1-n}\right)\int_n^1 y^d(2)^{\frac{\theta-1}{\theta}} d2\right]^{\frac{\theta}{\theta-1}}. \qquad \text{(A.29)}$$

Inserting the demand functions in place of $y^d(1)$ and $y^d(2)$, we obtain

$$Y^1 = Q^{1-n}(C + G), \; Y^2 = Q^{-n}(C + G). \quad \text{(A.30)}$$

Notice that while total consumption is completely insured, aggregate production can vary between regions through the terms of trade, Q.

Household's j intertemporal budget constraint reduces is as in (A.9)

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[\int_0^1 p_T(z)\, c_T^j(z)\, dz\right]\right\} \leq \sum_{T=t}^{\infty} E_t\left\{R_{t,T}\cdot\left[\left(1-\tau^i\right)p_T(j)\, y_T(j)\right]\right\} + B_t^j,$$
$$j \in J^i, \; i = 1, 2, \qquad \text{(A.31)}$$

and in symmetric equilibrium it can be written as

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[P_T C_T\right]\right\} = \sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[\left(1-\tau^i\right)P_T Y_T\right]\right\} + B_t^i,$$
$$j \in J^i, \; i = 1, 2 \qquad \text{(A.32)}$$

Summing over households one can obtain the union-wide intertemporal budget constraint

$$\sum_{T=t}^{\infty} E_t\left\{R_{t,T}\left[P_T \Delta_T\right]\right\} = B_t \qquad \text{(A.33)}$$

Where $\Delta_T \equiv \Delta_T^1 + \Delta_T^2$ is the consolidated primary deficit of the union and $B_t = B_t^1 + B_t^2$ is the aggregate public debt.

The price setting is very similar to that exposed in the simplified version and implies that the price indexes evolve according to

$$\begin{aligned} P_{1,T} &= \left[\alpha P_{1,T-1}^{1-\theta} + (1-\alpha) P_T(1)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \\ P_{2,T} &= \left[\alpha P_{2,T-1}^{1-\theta} + (1-\alpha) P_T(2)^{1-\theta}\right]^{\frac{1}{1-\theta}} \end{aligned} \tag{A.34}$$

The above is the set of conditions that must be satisfied by a rational expectation equilibrium, i.e. a sequence of $\{P_T, P_T^*(1), P_T^*(2), P_{1T}, P_{2T}, Y_{1T}, Y_{2T}, i_T, B_T\}_{T=t}^{\infty}$ given the exogenous sequence $\{\Delta_T\}_{T=t}^{\infty}$ of consolidated deficits of the two governments.

In this setup, in the absence of any constraint on fiscal policy, one can show that stochastic variations in the compounded budget of the two governments interfere both with price stability and with macroeconomic stability in the union. Furthermore this happens regardless of the form of the monetary policy, including the one proposed here where the central bank acts independently from the two fiscal authorities[24].

[24] See Woodford (1996) for a formal proof.

FIG. 1

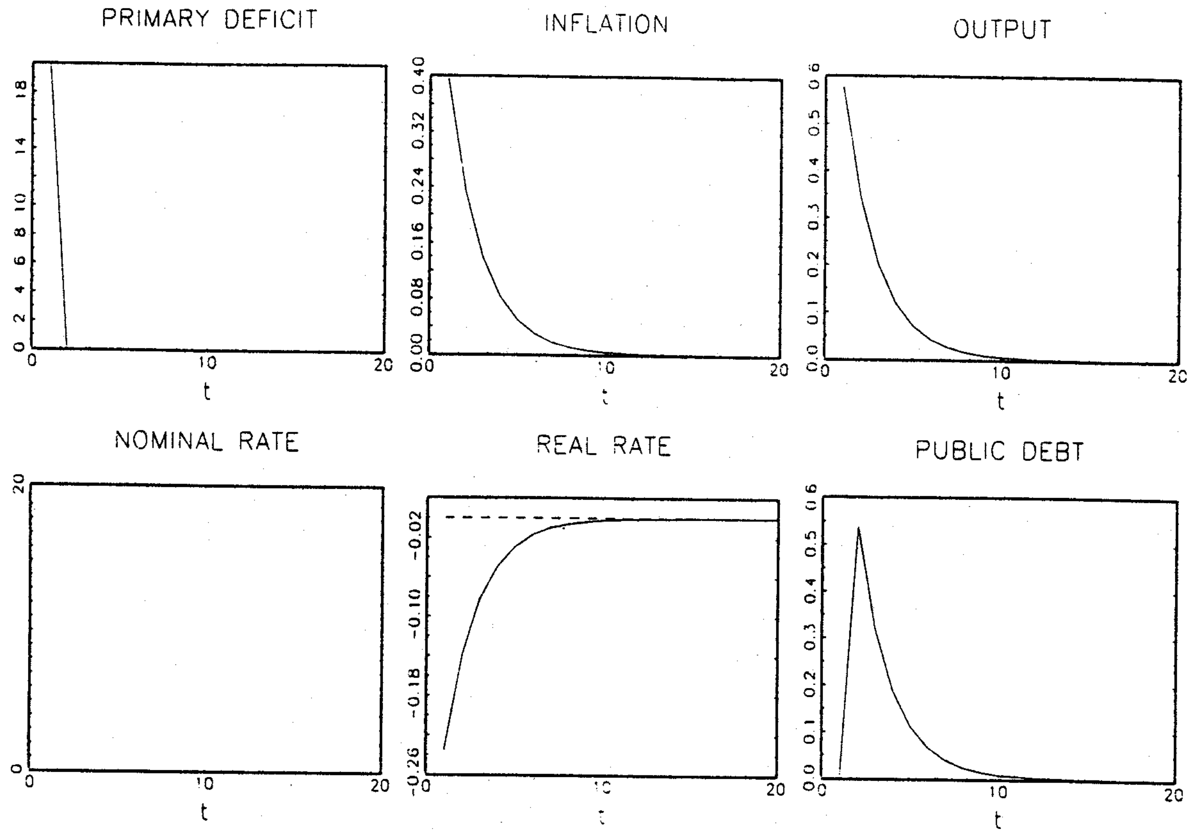

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FIG.2

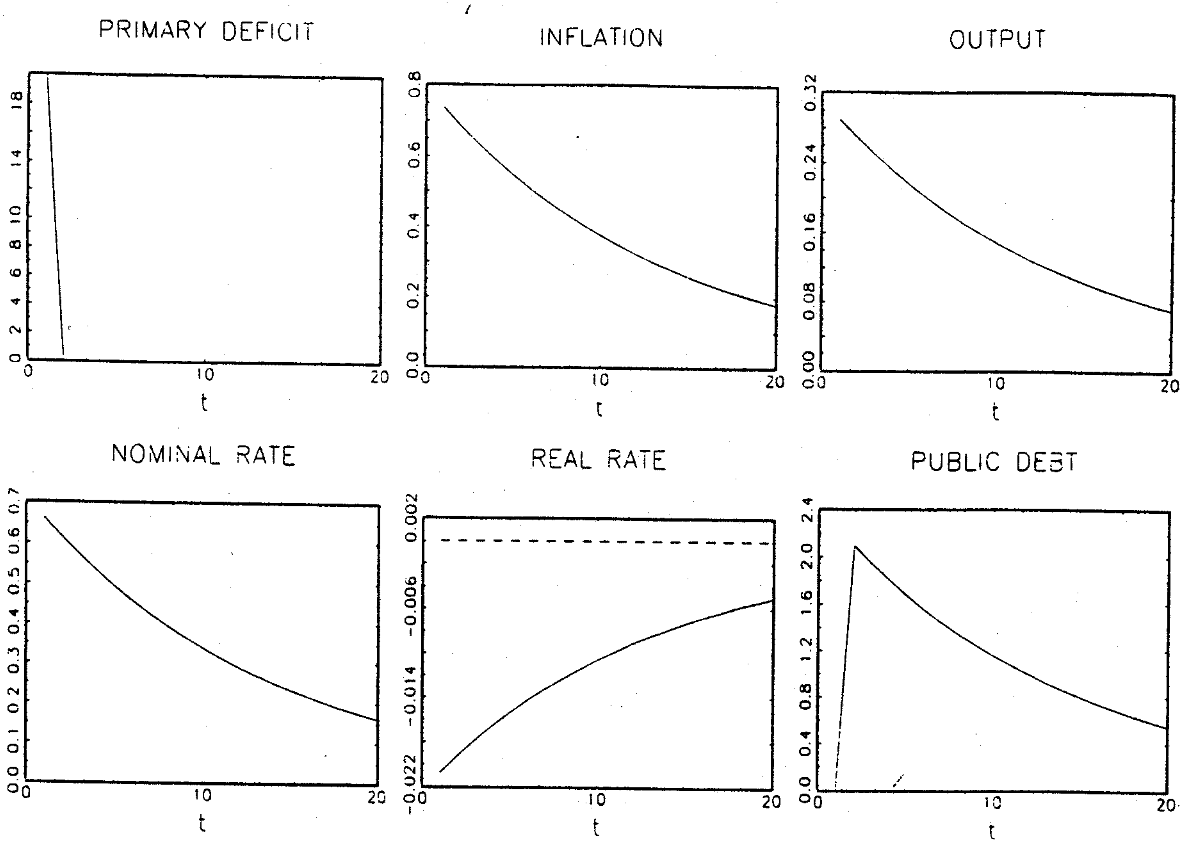
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FIG. 3

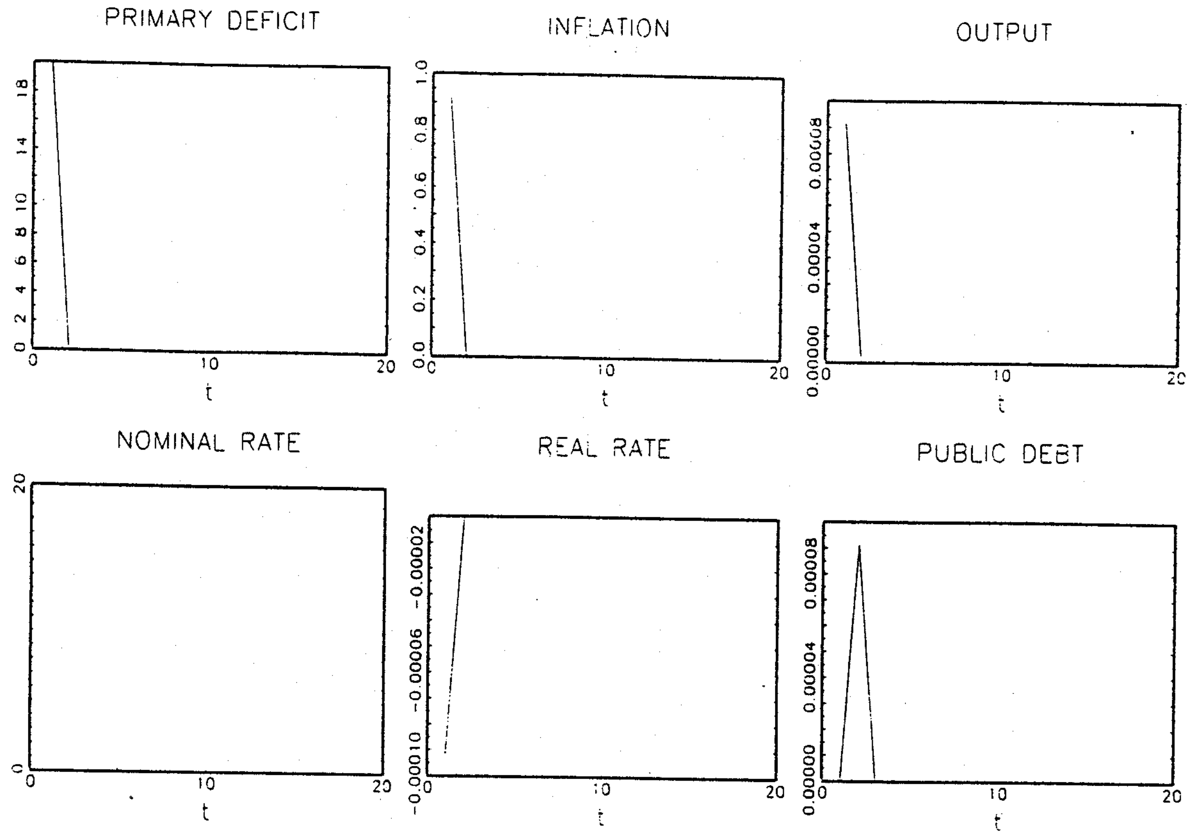

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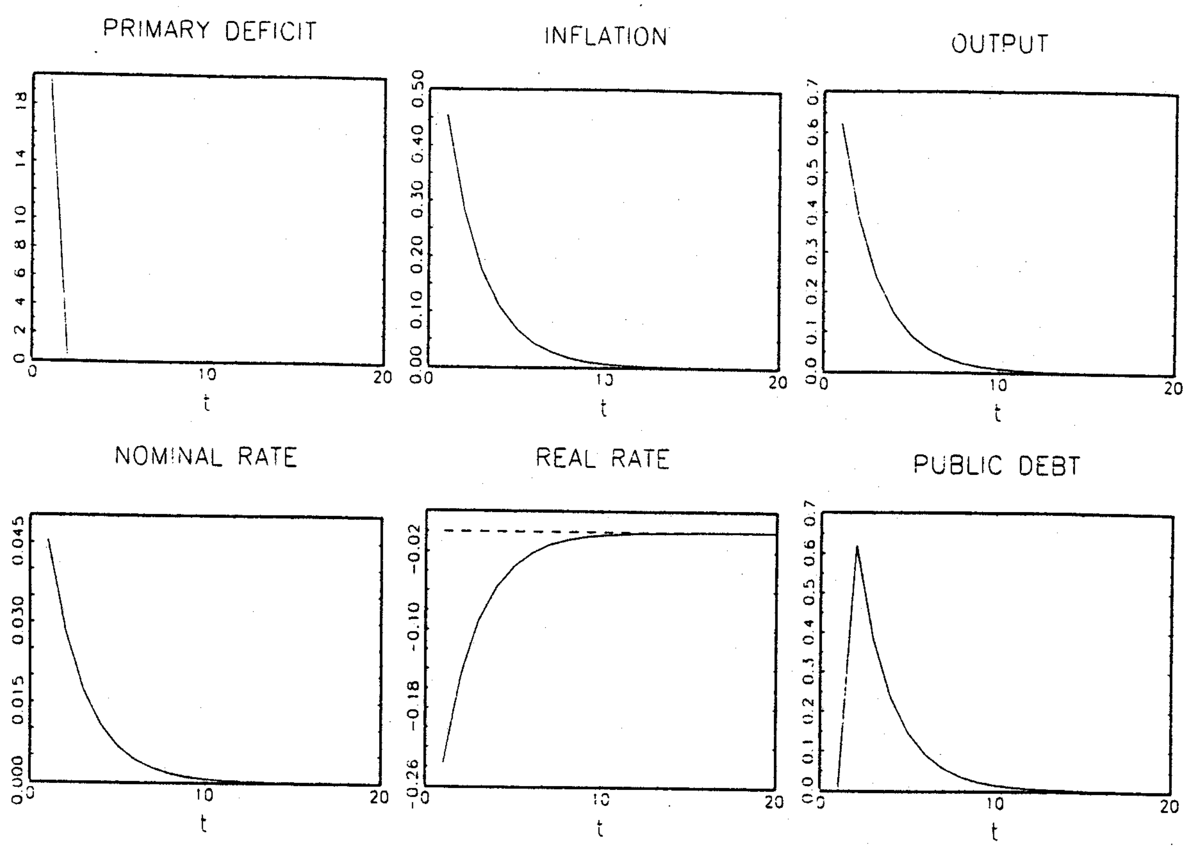
FIG. 4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