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COMMITTEE ON THE PEACEFUL USES
OF OUTER SPACE

STUDY ON ALTITUDES OF ARTIFICIAL EARTH SATELLITES

Working paper presented by the Secretariat

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Note by the Secretariat

The Committee on Peaceful Uses of Outer Space at its thirteenth session commented favourably on the need to ensure better co-ordination between its two Sub-Committees.

In this connexion, the Committee felt that its Scientific and Technical Sub-Committee could assist in the work of the Legal Sub-Committee by reviewing at an appropriate time its examination of criteria connected with the definition and/or delimitation of outer space, taking account of the contents of documents A/AC.105/39, of 6 September 1967, and A/AC.105/C.2/7, of 7 May 1970 (A/10020, para. 45).

In the report on its fifth session, held in 1967 (A/AC.105/39) the Scientific and Technical Sub-Committee adopted the following recommendation on the definition of outer space (para. 36):

> "The Sub-Committee in considering the request made by the Legal Sub-Committee of the Committee on the Peaceful Uses of Outer Space (A/AC.105/C.1/L.22) agreed as follows:
>
> "(a) That there was consensus in the Scientific and Technical Sub-Committee that it is not possible at the present time to identify scientific or technical criteria which would permit a precise and lasting definition of outer space;
>
> "(b) That the working papers prepared by the delegations of Canada and France, as well as the background paper prepared by the Outer Space Affairs Group of the United Nations Secretariat, and the relevant summary records of the Scientific and Technical Sub-Committee's meeting would be made available to the Legal Sub-Committee to assist its deliberations;
>
> "(c) That a definition of outer space, on whatever basis recommended, is likely to have important implications for the operational aspects of space research and exploration, and that it is therefore appropriate that the Scientific and Technical Sub-Committee continue consideration of this matter at future sessions; and that Member States be invited to submit further relevant material for the study of the Sub-Committee."

Document A/AC.105/C.2/7 is a background paper prepared by the Secretariat in May 1970 under the title: "The question of the definition and/or delimitation of outer space".

The Committee on the Peaceful Uses of Outer Space did not request the Secretariat to prepare any documents for the thirteenth session of the Scientific and Technical Sub-Committee and consequently no updating or revision of the background paper has been undertaken.

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Without prejudicing the definition of outer space to be based on any principle agreed upon by the Scientific and Technical Sub-Committee, it has been established that new technical data have accumulated on satellites orbits, both from theoretical considerations and from interpreted observations. These new data somewhat modify former conclusions as to the lowest perigee of an orbiting satellite and they are presented here for the convenience of the Sub-Committee.

The study has been prepared by the Working Group 1 of the Committee on Space Research (COSPAR) of the International Council of Scientific Unions and its authors are D. G. King-Hele, Royal Aircraft Establishment, Farnborough, Hants., England, L. Sehnal, P. Lála, J. Klokočnik, Astronomical Institute of the Czechoslovak Academy of Sciences, Ondřejov, Czechoslovakia. The study is presented here in annex I in a condensed form and issued in English only. Annex II, containing statistics of satellite perigee heights, prepared by the Secretariat, illustrates the main conclusions of the COSPAR study. For convenient reference and use by members of the Committee, the foreward by COSPAR and the introduction to the study are presented below in the four lanagues of the Committee.

Forward by COSPAR and introduction to the study

Forward

This study deals with the orbits of the artificial satellites of the Earth from the point of view of the altitudes in which they move above the surface of the Earth.

Special attention is paid to the conditions in the lowest altitudes in which the satellites move and to the disturbing forces which can affect the diminishing of the height of the closest point of approach of a satellite to the Earth.

The study is divided into five parts, dealing with the atmospherical effects (1), Earth gravitational field (2), lunisolar perburbations (3), solar radiation pressure and other minor effects (4). All this is supplemented by part (5) which deals with the ascent to, or descent from, the orbit.

The atmosphere is certainly the decisive phenomenon affecting the lifetime of a satellite and determining the character of the last parts of its orbit. The other disturbing effects contribute to the moment of time when the atmospherical drag becomes the predominant force causing the changes in satellite's orbit.

Thus, we tried to estimate the ratios of the different disturbing forces to drag and to describe the possible changes in perigee height; we concentrated on studying the upper limits.

However, the determination of the evolution of the satellite orbit is generally a very complex problem, which cannot be solved always with great accuracy. The extent of individual properties of special satellites and the time variability of the disturbing forces makes the problem difficult for any generalization.

It seems that the past estimates of the lowest heights into which satellites can plunge, without falling down to the ground or burning up in the atmosphere, were too high. This is especially true for satellites with highly eccentric orbits which penetrate into the atmosphere for a limited time during each revolution around the Earth.

Introduction

The orbit of a satellite can be fully described by the so-called orbital elements. They are referred to a special reference frame illustrated in the figure below.

The basic reference frame is given by the centre of the Earth (E), the equatorial plane and by a fixed geocentric direction which is the crossing of the equator with the plane of the ecliptic. This direction indicates a point on the celestial sphere, the "vernal equinox".

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Some necessary notions are to be mentioned: the point of the orbit which is closest to the geocentre is the perigee; the farthest point is the apogee. The distance of the perigee from the geocentre is the radius-vector of the perigee, whereas the height of the perigee means its distance from the surface of the Earth.

The angle Ω is the longitude of the ascending node, ω is the argument of perigee, i is the inclination of the orbit. Those three elements describe the position of the orbit in space. The distance AP is twice the semi-major axis a, the distance EP is $a\ (1-e)$ where e is the eccentricity of the orbit. The latter two elements describe the size and shape of the orbit. The position of the satellite in the orbit is defined by the time of the perigee passage of the satellite (T_o). Those six reference values (Ω, ω , i, a, e, T_o) describe fully the orbit of the satellite and its position in a geocentric reference frame. They are referred to as "Keplerian elements".

In the absence of any other body except the satellite and the Earth, and if the Earth were homogeneous, rigid and spherical, and possessed no atmosphere, the orbit of the satellite, as described by the Keplerian elements, would be stable in space and constant in shape. But, since the above assumptions are not fulfilled, the Keplerian elements are changing in time and we say that they are affected by perturbations. A perturbation is called secular if it steadily increases with time. Short periodic perturbations have periods not longer than one or a few revolutions of a satellite; long periodic perturbations have substantially longer periods. All those perturbations are then included in the definition of the orbit in time-dependent terms.

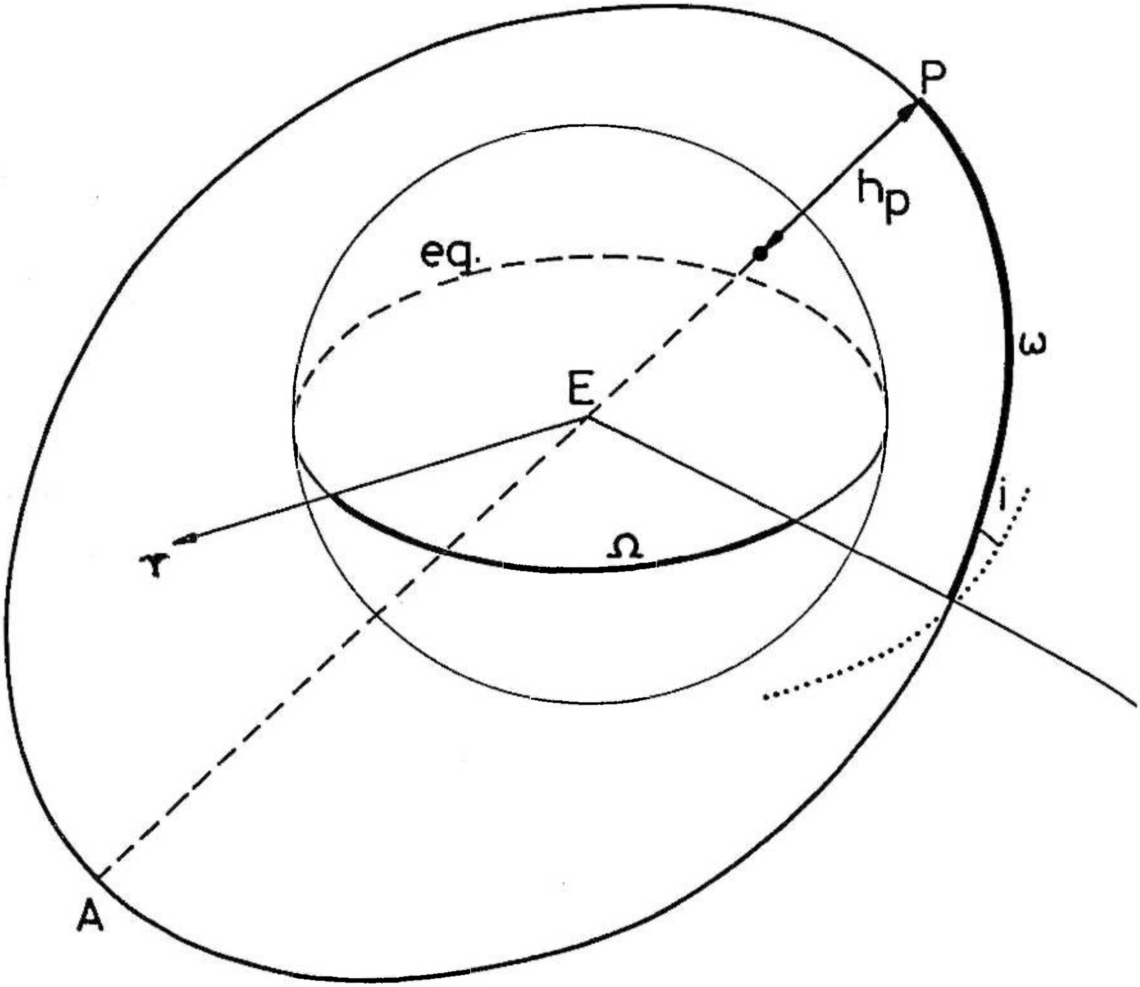

Figure I.

The definition of orbital elements
P - perigee, A - apogee, E - center of the Earth,
♈ - vernal equinox, eq. - equator of the Earth,
h_p - perigee height, Ω - longitude of ascending node,
ω - argument of perigee, i - inclination

Annex I

STUDY ON ALTITUDES OF ARTIFICIAL EARTH SATELLITES

Part 1

The Earth's atmosphere effects

1. Introduction

If the orbit of an Earth satellite comes within 1,000 km of the Earth's surface, the satellite suffers an appreciable aerodynamic drag at its perigee. The effect of the drag is to reduce the speed of the satellite at perigee, and as a result the satellite does not fly out to such a great distance at the other side of the Earth. Its maximum height - at the apogee point - decreases and the orbit becomes more nearly circular, as shown in figure 1. Eventually, usually after a number of years if the perigee is above 300 km, or only a few weeks if the perigee is below 150 km, the satellite loses height catastrophically, encounters fierce aerodynamic heating and plunges to fiery destruction with its energy of motion converted to heat in the relatively dense atmosphere at heights below 90 km.

This study concentrates on orbits with perigee heights below 150 km, for two reasons. First, satellites with high perigees remain in orbit for many years, little affected by the atmosphere. Second, the density of the upper atmosphere varies greatly between day and night, and is also extremely sensitive to variations in the ultra-violet radiation from the Sun, at heights above 150 km: this makes it impracticable to present useful results for satellites above 150 km, without a very large number of graphs. Below 150 km the atmospheric density does not usually vary by more than 20 per cent as a result of solar and geophysical phenomena.

2. The decrease of density with height

Figure 2 gives the variation of air density with height between 90 and 150 km height, from reference 1. The air density is expressed in milligrams per cubic metre, and figure 2 shows that it decreases from 3.4 mg/m^3 at 90 km height, to 0.002 mg/m^3 at 150 km height. In other words, the density is nearly 2,000 times greater at 90 km than at 150 km, so that great differences in the effect of the atmosphere on an orbit must be expected if a satellite's perigee height is 90 km rather than 150 km. Figure 2 also shows that the density falls off most rapidly in the region between 90 and 120 km, with a more gradual decrease at higher altitudes.

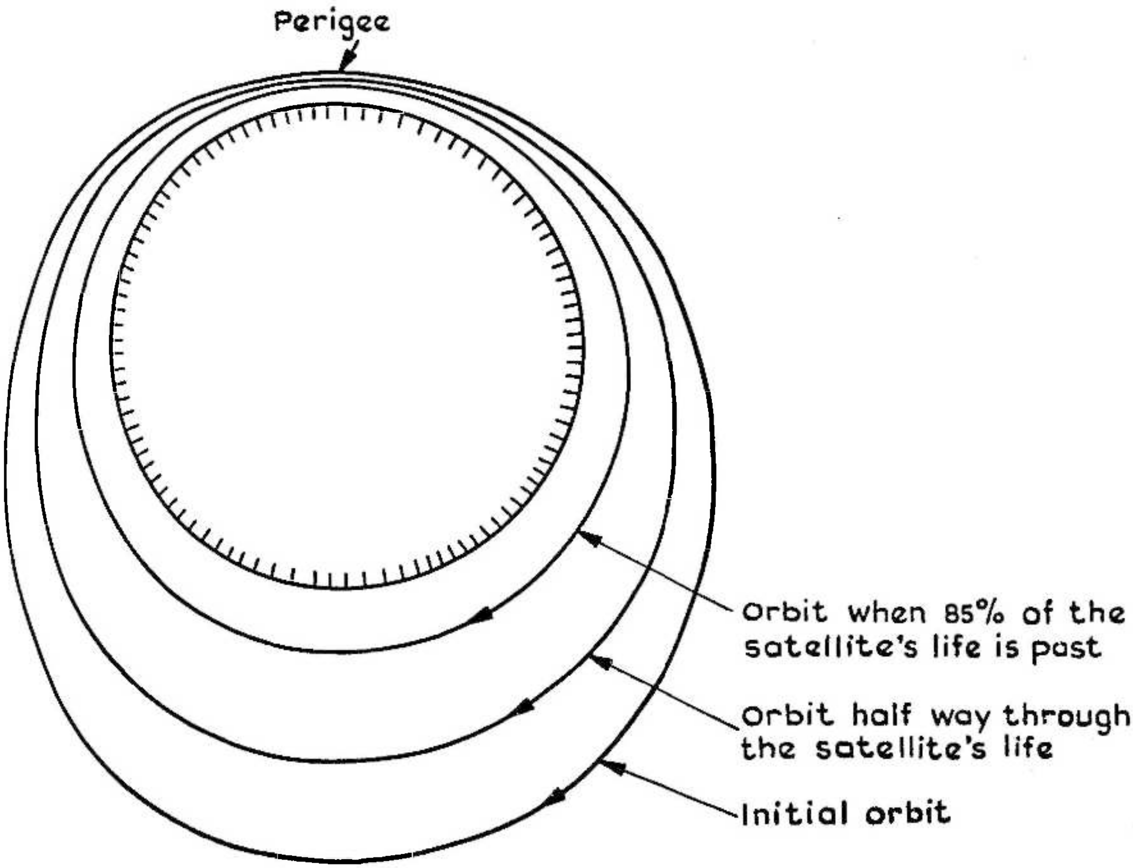

Fig.1 Contraction of satellite orbit under the action of air drag

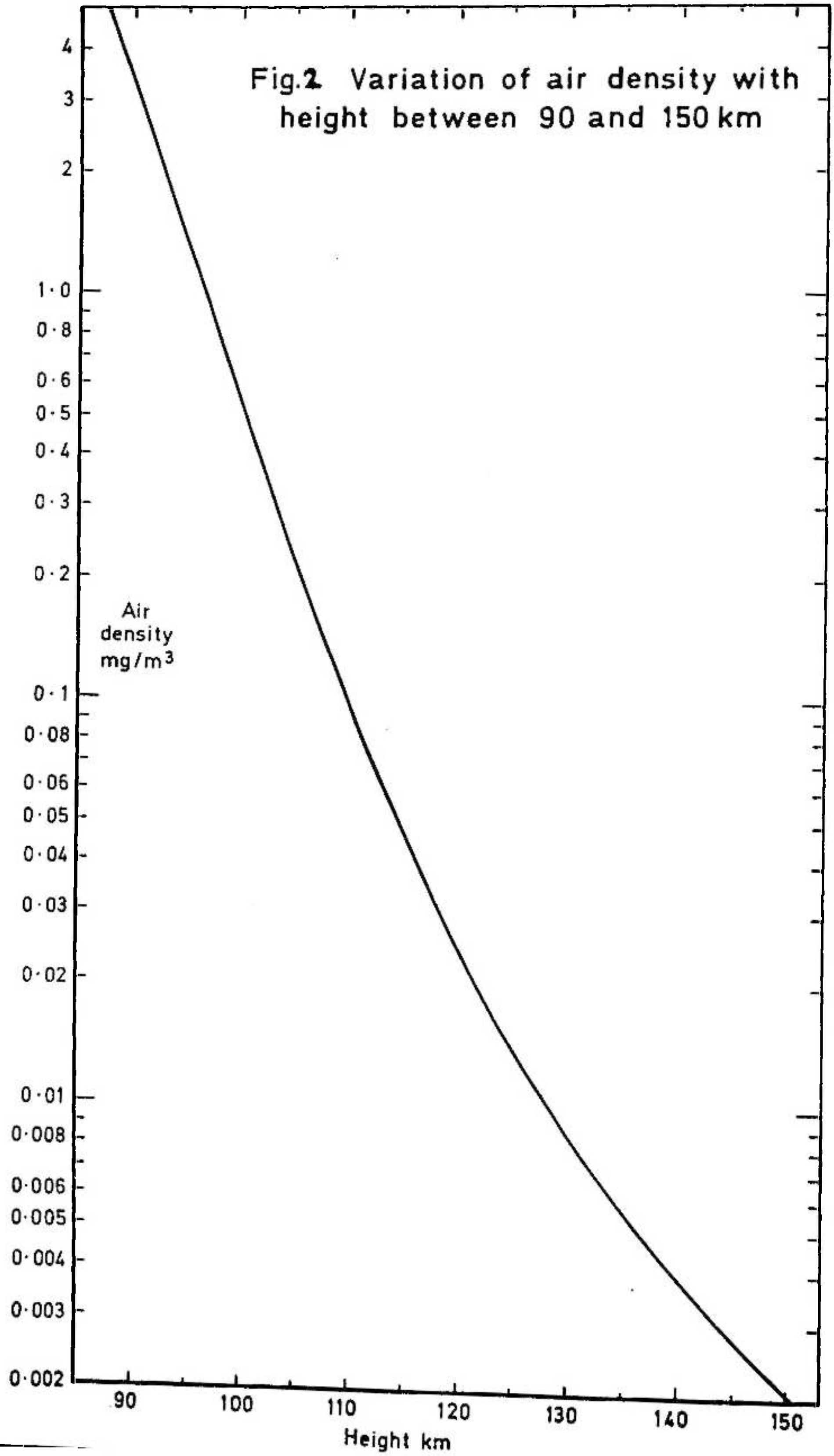
Fig.2 Variation of air density with height between 90 and 150 km
4
3
2
1·0
0·8
0·6
0·5
0·4
0·3
0·2
Air density mg/m3
0·1
0·08
0·06
0·05
0·04
0·03
0·02
0·01
0·008
0·006
0·005
0·004
0·003
0·002
90
100
110
120
130
140
150
Height km

3. Defining the lifetimes of satellites

If the variation of density with height is specified, as in figure 2, it is possible to derive formulae (given in the appendix), for the lifetime of a satellite in terms of (a) its perigee height, (b) the eccentricity of the orbit, and (c) the satellite's mass/area ratio (see section 4).

The eccentricity is a measure of how much the orbit departs from a circle, and the maximum (apogee) height for any particular eccentricity is shown in figure 3. Thus an eccentricity of 0.1 corresponds to an apogee height near 1,500 km, and an eccentricity of 0.5 corresponds to an apogee height near 13,000 km.

4. The mass/area ratio

The mass/area ratio of a satellite is defined as its mass divided by the effective cross-sectional area which it presents as it moves through the air. For a spherical satellite, this area is just πr_s^2, where r_s is its radius; for a tumbling cylindrical satellite, the appropriate area is a little less than the length multiplied by the diameter.

Figure 4 shows the mass/area ratios for examples of 20 different types of actual satellites representative of the 1,500 satellite launches that had been made by August 1975. The names and lengths in metres of the satellites given in figure 4 are listed in table 1. In figure 4 the values of mass/area are plotted against the mass of the satellite, which ranges between 10 kg and 56,000 kg. The heavier the satellite, the more likely it is that some pieces will survive atmospheric re-entry during its final decay.

Figure 4 shows that the mass-to-area ratio is generally between 50 and 150 kg/m^2, though the values are quite widely scattered, between 30 and 250 kg/m^2.

The lifetime of a satellite of a given size is proportional to its mass/area ratio: if the satellite was loaded up with lead weights to double its mass, its lifetime (for a given orbit) would be doubled. So it is only necessary to give lifetimes for one representative value of mass/area ratio, and 100 kg/m^2 has been chosen here as the most suitable average value.

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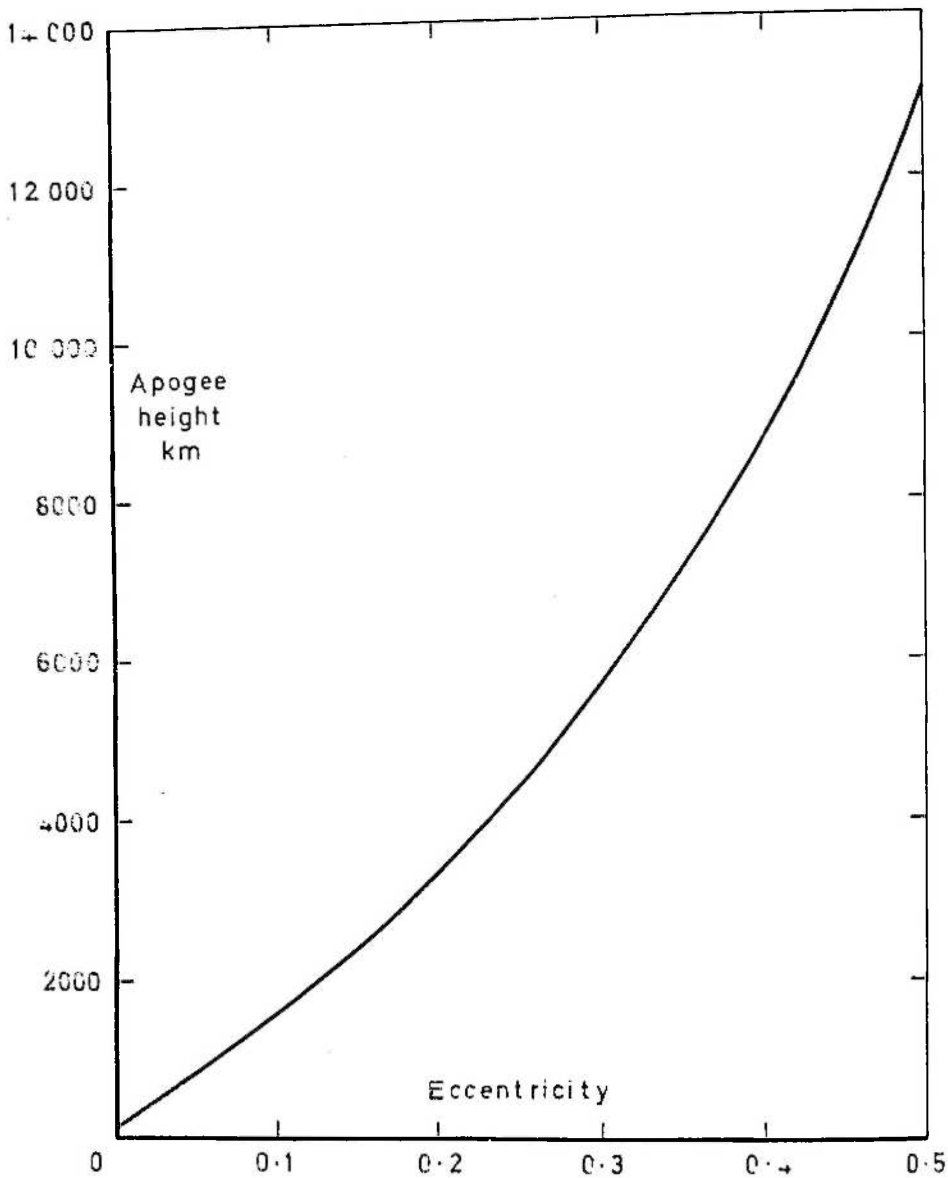

Fig. 3 Variation of apogee height with eccentricity for orbits with perigee height near 120 km

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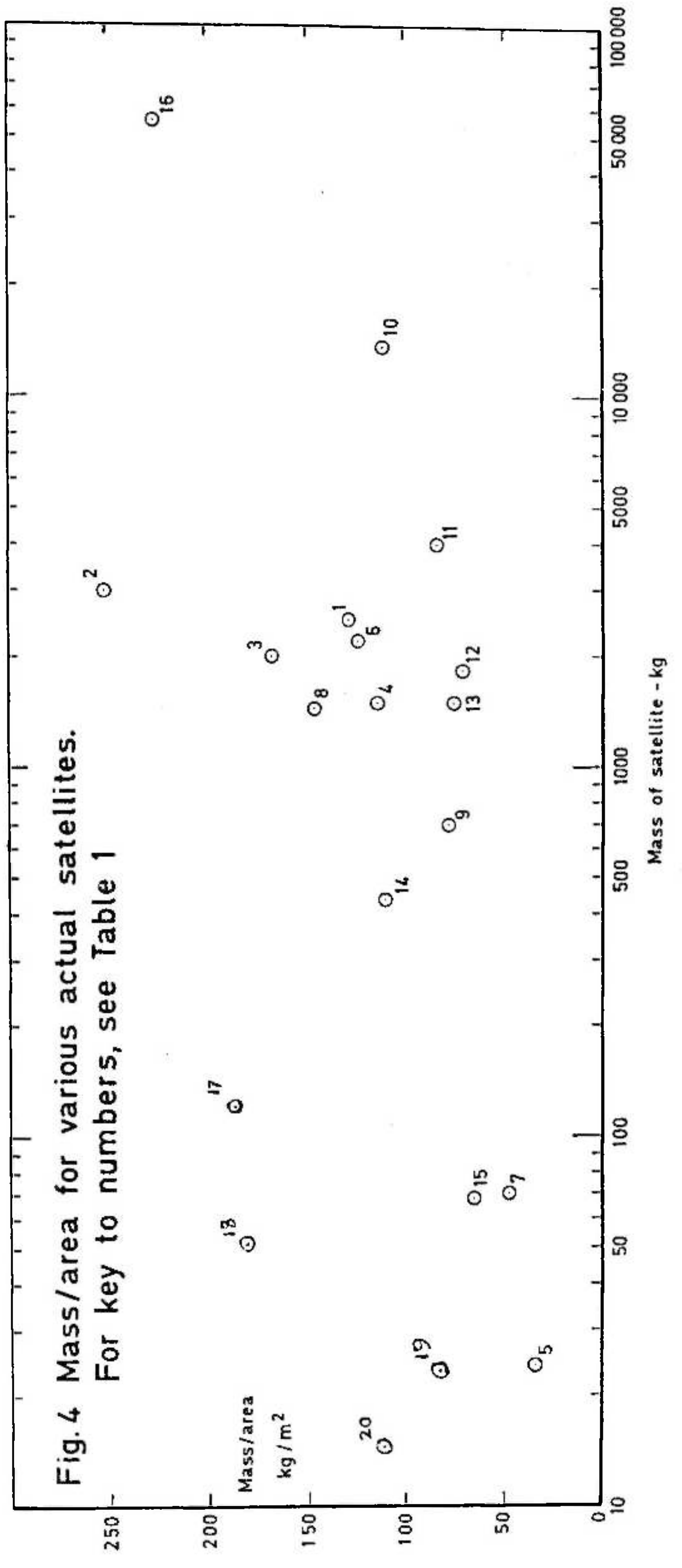

Fig. 4 Mass/area for various actual satellites. For key to numbers, see Table 1

Table 1 Key to the numbers on Fig 4

Number on Fig 4	Name of satellite		Length, m
1	1969-08A	Cosmos 264 rocket	7.5
2	1969-19A	Titan 3B-Agena D	8.0
3	1969-26A	Thorad Agena D	8.0
4	1969-33B	Cosmos 277 rocket	8.0
5	1969-16B	Essa 9 rocket	1.50
6	1969-24B	Cosmos 272 rocket	7.4
7	1969-25E	OV1-19 rocket	2.05
8	1969-84B	Meteor 2 rocket	3.8
9	1970-25C 1969-82AB	Nimbus 4 rocket Agena D rocket	6.0
10	1969-43B	Saturn IVB	18.7
11	1969-58B 1968-103B	Luna 15 launcher rocket Proton 4 rocket	12
12	1969-69B	ATS 5 rocket	8.6
13	1969-77B	Cosmos 298 rocket	8.0
14	1970-13D	Molniya IN rocket	2.0
15	1970-109B	Peole 1 rocket	1.60
16	1973-27B	Skylab 1 rocket	24.8
17	1960-γ2	Transit 1B	0.91*
18	1962-βτ2	Injun 3	0.61*
19	1962-βτ5	[Thor Agena D]	0.6*
20	1965-65E	Tempsat 1	0.36*

* These are spherical satellites: the value given is the diameter.

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5. Graph of lifetimes

Figure 5 shows how the lifetime of a satellite with the standard mass/area ratio of 100 kg/m^2 varies with perigee height for values of eccentricity up to 0.5 (if the eccentricity exceeds 0.5, lunisolar perturbations rather than air drag are likely to dominate the decay behaviour). It should be remembered that the lifetime is proportional to the mass/area: if it is 50 km/m^2 rather than 100 kg/m^2, the lifetime will be half the value shown in figure 5.

The orbital period of a satellite always exceeds 86 minutes, or 0.06 day. The lifetime in figure 5 is therefore cut off at 0.06 day: if the lifetime is shorter than this, the satellite does not survive a complete orbit around the Earth, and so scarcely deserves the name of "satellite".

Figure 5 shows the wide variation in lifetime when the perigee height and eccentricity vary. When the perigee height is 150 km, the lifetime is only a little over one revolution if the orbit is circular, e = 0; even a small increase in eccentricity, to 0.01, increases the lifetime to 0.8 day, or 13 revolutions; and increasing e to 0.3 increases the lifetime to over 100 days. For a fixed eccentricity, say 0.1, the variation of lifetime with perigee height is very sharp: the lifetime is 24 days for a perigee height of 150 km, but less than one revolution for a perigee height of 90 km.

Most satellites decay in orbits of low eccentricity, because the eccentricity has been gradually destroyed during the life by the process shown in figure 1. But it is possible for orbits of high eccentricity, say 0.5, to be perturbed in such a way that the perigee is driven below 100 km before air drag has much chance to take effect. So there will be occasions when the curves on the left of figure 5 are relevant, although most satellites will have low values of e at decay.

It will be noticed that all the curves in figure 5 have a sharp bend at heights between 120 and 130 km. This irregularity occurs because the aerodynamic flow around a satellite changes its character completely as the atmosphere becomes denser. For heights above 150 km free-molecule flow prevails: the distance travelled by the air molecules between collisions exceeds 40 m, and is considerably greater than the dimensions of any normal satellite. In these conditions the molecules interact individually with the satellite, and do not interfere with each other. For satellites not exceeding 10 m in length, this situation prevails down to 130 km, where the mean free path is about 10 m. Then as the height decreases from 130 to 120 km, the satellite dimensions become greater than the mean free path (which is 3 m at 120 km), and the draft coefficient of the satellite is reduced to about half its free-molecule value (see reference 2). The lifetime for 120 km perigee height is therefore double what would be expected by extending downwards the curve appropriate for heights of 130-150 km. For smaller satellites, the dimensions become equal to the free mean path at a lower height, and the "bend" occurs lower: for example, a satellite of length 1 m has the bend between 110 and 100 km, as the broken lines in figure 5 show. The lifetimes of satellites of intermediate sizes will lie between the unbroken and broken curves in figure 5 for perigee heights of 100-130 km.

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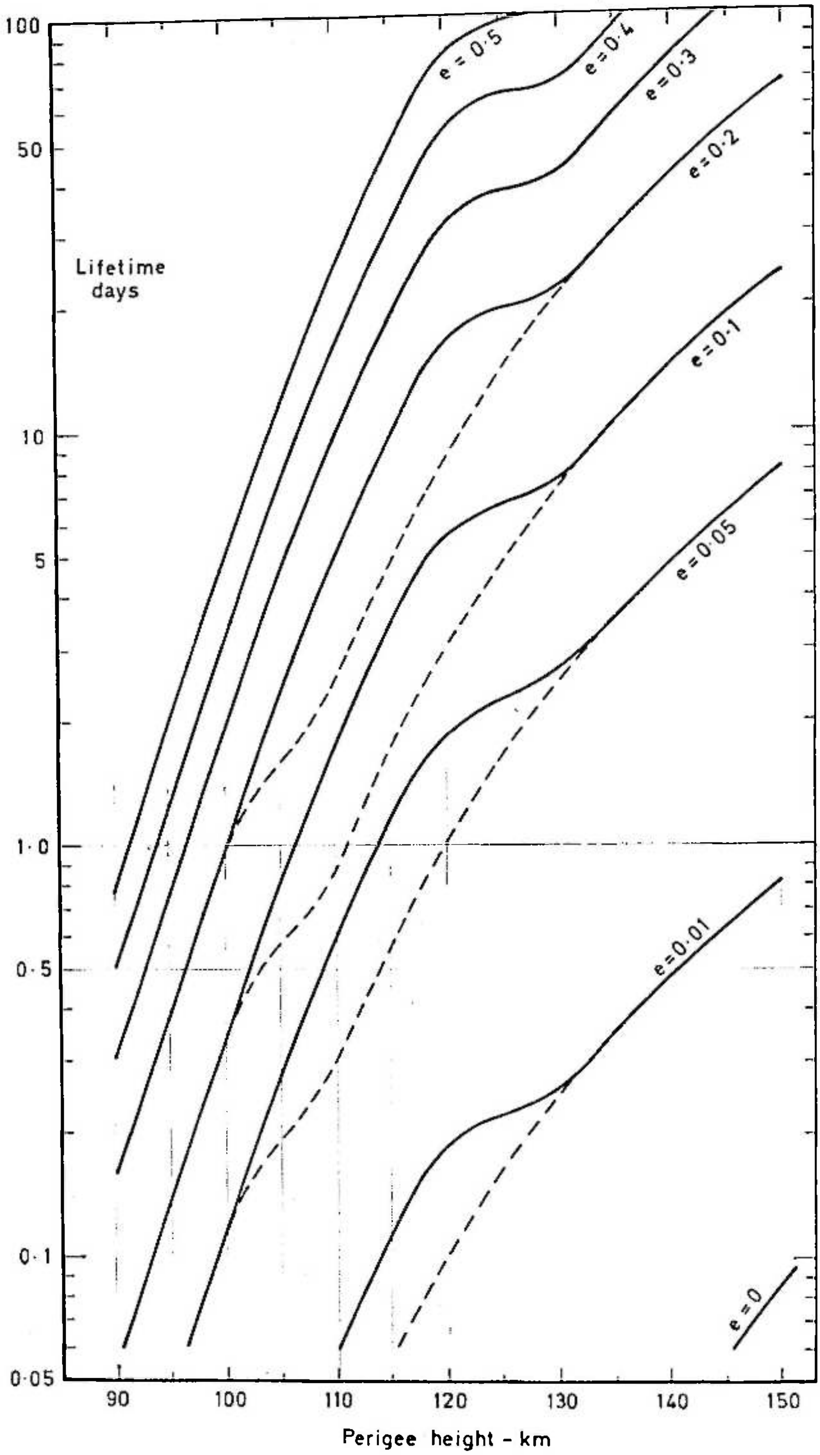

Fig.5 Lifetimes of satellites with mass/area 100 kg/m^2 (Unbroken curves for satellites of length 10 m : broken curves for length 1 m)

6. Conclusion

This paper gives the lifetimes of satellites with perigee heights between 90 and 150 km, and eccentricities between 0 and 0.5, see figure 5. These results are for satellites of mass/area 100 kg/m^2, and the lifetime is proportional to the mass/area, so that a satellite with 50 kg/m^2 would have half the lifetime shown in figure 5.

Appendix to part 1: Calculation of lifetime

If $e > 3H/a$, where H is the density scale height at perigee and a is the semi-major axis, equation (7.27) of reference 3 gives the air density ρ_A at height 1/2 x H above perigee, for a satellite of mass m and cross-sectional area S, as

$$\rho_A = - \frac{0.590 \dot{T}}{\delta\sqrt{aH}} E(e) \qquad (1)$$

where $\dot{T}$ is the rate of change of orbital period T; E(e) is a function of e given in figure 7.1 of reference 3; and $\delta = F S C_D/m$ where $F C_D$ may be taken as 2.2 for heights above 130 km. Since $H < 18$ km for perigee heights less than 150 km, equation applies for $e > 0.008$.

The lifetime L of a satellite, again for $e > 3H/a$, may be written

$$L = - \frac{e\,T\,F(e)}{\dot{T}} \qquad (2)$$

where F(e) is given by figure 4.10 of reference 3. Also we have

$$T = 2\pi\sqrt{a^3/\mu} \qquad (3)$$

where $\mu = 398.6 \times 10^{12}\ m^3/sec^2$, by Kepler's law, and, since the perigee distance $a(1-e)$ is 6490 ± 40 km,

$$a = \frac{6490}{1-e}\ \text{km} \qquad (4)$$

On taking $\delta = 2.2$ S/m and substituting (1), (3) and (4) into (2), we have

$$L = \frac{m}{S}\ \frac{Q(e)}{10^6\ \rho_A\ \sqrt{H}}\ \text{days} \qquad (5)$$

where m is in kg, S in m^2, ρ_A in kg/m^3 and H is in m, and

$$Q(e) = \frac{6.35\ e\ F(e)\ E(e)}{1-e} \qquad (6)$$

Figure 5 has been drawn using equation (5), with the values of $\rho_A \times \sqrt{H}$ from figure 1 and $m/S = 100\ kg/m^2$. For heights of 120 km and below, δ is taken as 1.1 S/m, thus doubling the lifetimes. For circular orbits,

$$L = \frac{m}{S}\ \frac{H}{10^{16}\ \rho}\ \text{days}. \qquad (7)$$

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Part 2

The gravitational field of the Earth

Provided there would be no atmosphere and no other bodies present, the orbit of an artificial satellite of the Earth would be a regular conic with stable position, if we could consider the Earth as a homogeneous sphere. However, the Earth is neither homogeneous nor spherical; its shape can be represented by a special surface with constant gravity potential, called "geoid". It shape corresponds approximately to the surface of the oceans.

The shape of the Earth (and also its gravitational field in which the satellites move) can be mathematically described by a series of terms where the different anomalies are represented by numerical values of the "harmonic coefficients". The largest coefficient J_2 corresponds to the polar flattening, the coefficient J_3 characterizes the assymetry with respect to the equator, and higher coefficients describe the shape of the Earth in still more detail. At present, we know numerically about 400 harmonic coefficients. The coefficient J_2 (flattening of the Earth) is by about a factor of 1,000 greater than the others.

The polar flattening of the Earth makes a difference of about 20 km between the equatorial and polar radius of the Earth. Thus, the height of perigee for similar orbits may differ with respect to the latitude of the position of the perigee.

The coefficient J_2 causes secular changes in the position of the orbit. The magnitude of this change reaches several minutes of arc per revolution of the satellite so that the orbit may fully rotate within a month. On the other hand, there are no secular changes in the semi-major axis and eccentricity. Thus, the most conspicuous effect caused by the irregularity of the shape of the Earth is the change of the position of the orbit in space, whereas its shape and dimensions suffer only periodical changes. The latter is true also for the height of the perigee - this may cause only minor changes in the lifetime of the satellite.

Let us now discuss in more detail the principal perturbations of the perigee height, arising from the main features of the geoid.

Large long periodic changes arise from the effects of coefficient J_3, which describes the unevenness of the northern and southern hemispheres. Table 2 gives numerical values for some cases.

The irregular shape of the Earth is characterized also by the deviation of the equator from a circle. As the Earth rotates, the irregularities change their position rapidly with respect to the satellite's orbit. If the rotation of the Earth is in resonance with the revolution of the satellite, resonant long periodic changes arise with significant amplitudes. During a resonance, the eccentricity of the orbit changes by about 0.0003 in 100 days. The changes of the perigee height can reach $\pm$ 2 km, which is less than the effects of the odd harmonics (J_3). The conditions of resonance are sooner or later destroyed by other disturbing effects (atmosphere); the usual resonant interval is of 100 days.

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Table 2

Long periodic changes of perigee height due to J_2 and J_3 coefficients of the Earth gravitational field.

i (deg)	J_2		J_3	
	ampl. (km)	period (days)	ampl. (km)	period (days)
0	0.000	-	0.00	-
20	0.008	12	2.57	23
40	0.032	21	4.82	41
60	0.164	159	6.49	318
80	0.012	47	7.39	94
90	0.020	40	7.50	80

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Part 3

Lunisolar gravitational perturbations of satellite orbits

Lunisolar perturbations are caused by the grativational attraction of the Moon and Sun on an Earth satellite. It can be shown that the gravitational influence of other bodies of the solar system is very small as compared to that of the Sun and the Moon.

The perturbations caused both by the Moon and Sun can be treated by the same method; from the point of view of a geocentric motion of an Earth satellite, both disturbing bodies differ only in their masses and orbits. We consider the orbits of Moon and Sun to be circular, the masses and orbital radii given as:

$m_{☽} = 7.35 \times 10^{22}$ kg $r_{☽} = 384\ 403.12$ km

$m_{\odot} = 1.99 \times 10^{30}$ kg $r_{\odot} = 149\ 480\ 000$ km

Despite the large mass of the Sun, the Moon exerts generally greater (about a factor of 2) disturbances, owing to its smaller distance.

The perigee height of an Earth satellite is influenced through a change of the orbital eccentricity. The semi-major axis has neither secular nor long-periodic changes. Thus, since

$$R_{\otimes} + h_p = a\,(1 - e)$$

we have

$$\Delta h_p = -\ a\Delta e$$

Here h_p is the height of the perigee, a the semi-major axis, e the eccentricity, and $R_{\otimes}$ the radius of the Earth.

The magnitude of the lunisolar perturbations obviously increases with satellite's distance from the Earth. Considering two orbits with the same initial perigee height, the change of the perigee height is greater for the orbit coming closer to the Moon (i.e. for the orbit with larger eccentricity, larger apogee distance). For the variation of apogee height with eccentricity see figure 3.

The atmospherical drag has obviously the decisive role on the final height in which the satellite moves before decay. However, the lunisolar effects can diminish the perigee distance so that the satellite penetrates earlier into the dense layers of the atmosphere.

As an example, we recall the orbit of the third Soviet lunar probe, Luna 3, 1959 01, which was launched on 4 October 1959, and which took the first photographs of the lunar far side. The probe came as close as about 8 000 km to the Moon. The lunar perturbations caused a rapid decrease of the perigee height so that the probe ended its life after its eleventh revolution in the Earth-Moon system.

In a geocentric co-ordinate system, the motion of a body with negligible mass (artificial satellite) can be described as

$$\ddot{\bar{r}} = -\mu \frac{\bar{r}}{r^3} + \mu_D \left(\frac{\bar{r}_D - \bar{r}}{(r_D - r)^3} - \frac{\bar{r}_D}{r_D^3} \right) \qquad (8)$$

The first term on the right-hand side corresponds to the undisturbed motion around a central body. The second term describes the disturbing acceleration caused by a third body (either the Moon or the Sun).

The magnitude of disturbing accelerations caused by the Moon or the Sun for different heights above a spherical Earth are summarized in Table 3, which shows also, for the sake of comparison, the ratio of lunar and solar disturbing accelerations to that of the asphericity of the Earth.

Table 3

The magnitudes of disturbing acceleration of the moon ($F_{☽}$) and of the sun ($F_{\odot}$) for different heights above a spherical Earth. The ratios of the lunar and solar disturbing accelerations to that due to asphericity of the Earth (F_{flat}) are shown.

h (km)	$F_{☽}$ m/sec^2 x 10^{-6}	$F_{☽}/F_{flat}$	$F_{\odot}$ m/sec^2 x 10^{-6}	$F_{\odot}/F_{flat}$
100	1.15	0.00004	0.51	0.00002
200	1.16	0.00004	0.52	0.00002
500	1.21	0.00005	0.55	0.00002
1 000	1.31	0.00007	0.59	0.00003
1 500	1.40	0.00010	0.62	0.00005
2 000	1.49	0.00014	0.66	0.00006
3 000	1.68	0.00025	0.74	0.00011
5 000	2.05	0.00065	0.90	0.00029
10 000	3.00	0.0041	1.30	0.0018
20 000	5.01	0.046	2.09	0.019
30 000	7.16	0.24	2.89	0.096
50 000	11.85	2.27	4.47	0.86
70 000	17.08	11.0	6.06	4.26
100 000	25.93	63.0	8.45	20.5

The lunar disturbing acceleration is more than twice that of the sun - this fact is reflected also in the values of the changes of elements.

At low altitudes, the ratio of the lunisolar disturbing acceleration to that of the atmospherical drag rapidly diminishes. The drag depends not only on the mass of the satellite but also on its effective cross-section (see part 1). On the other hand, the density of the atmosphere depends on the exospheric temperature which is a function of solar activity.

The disturbing acceleration caused by the drag is given by

$$F_{drag} = - \frac{1}{2} C_D \frac{S}{m} \rho V^2 , \qquad (9)$$

where C_D is the aerodynamical coefficient, δ is the density of the atmosphere, and V the velocity of the satellite. For a circular orbit with radius r, we have

$$V^2 = \frac{\mu}{r} .$$

The altitude where the disturbing accelerations caused by lunisolar effects and drag are in balance, can be derived from eq. (7) and (8). The respective altitudes follow from

$$\frac{(h + R)^2}{\rho \ (h)} = \frac{1}{4} \frac{m_{\oplus}}{m_D} r_D^3 \ C_D \frac{S}{m} \qquad (10)$$

where $m_{\oplus}$, m_D , m are the masses of the Earth, the disturbing body and the satellite respectively. The results are plotted in figure 6, for different exospheric temperatures. The heights depend on the S/m ratio, which is given in exponential scale.

The changes of elements caused by the lunisolar gravitational attraction can be computed from the basic equation (8).

Omitting insignificant short-periodic perturbations and supposing that the orbit of the satellite and the position of the disturbing body do not change during one revolution of the satellite, we see that the change of the semi-major axis vanishes and the period of revolution of the satellite thus remains unaffected.

The change in perigee height, h_p, can be computed from the equation:

$$\Delta h_p = \frac{15}{2} \pi \frac{\mu D}{\mu} \left(\frac{a}{r_D} \right)^3 a \ e \sqrt{1-e^2} \ \cos i' \cos 2 \omega' \qquad (11)$$

where i' and ω' are the inclination and the argument of perigee of the satellite orbit, measured in the orbital reference frame of the disturbing body.

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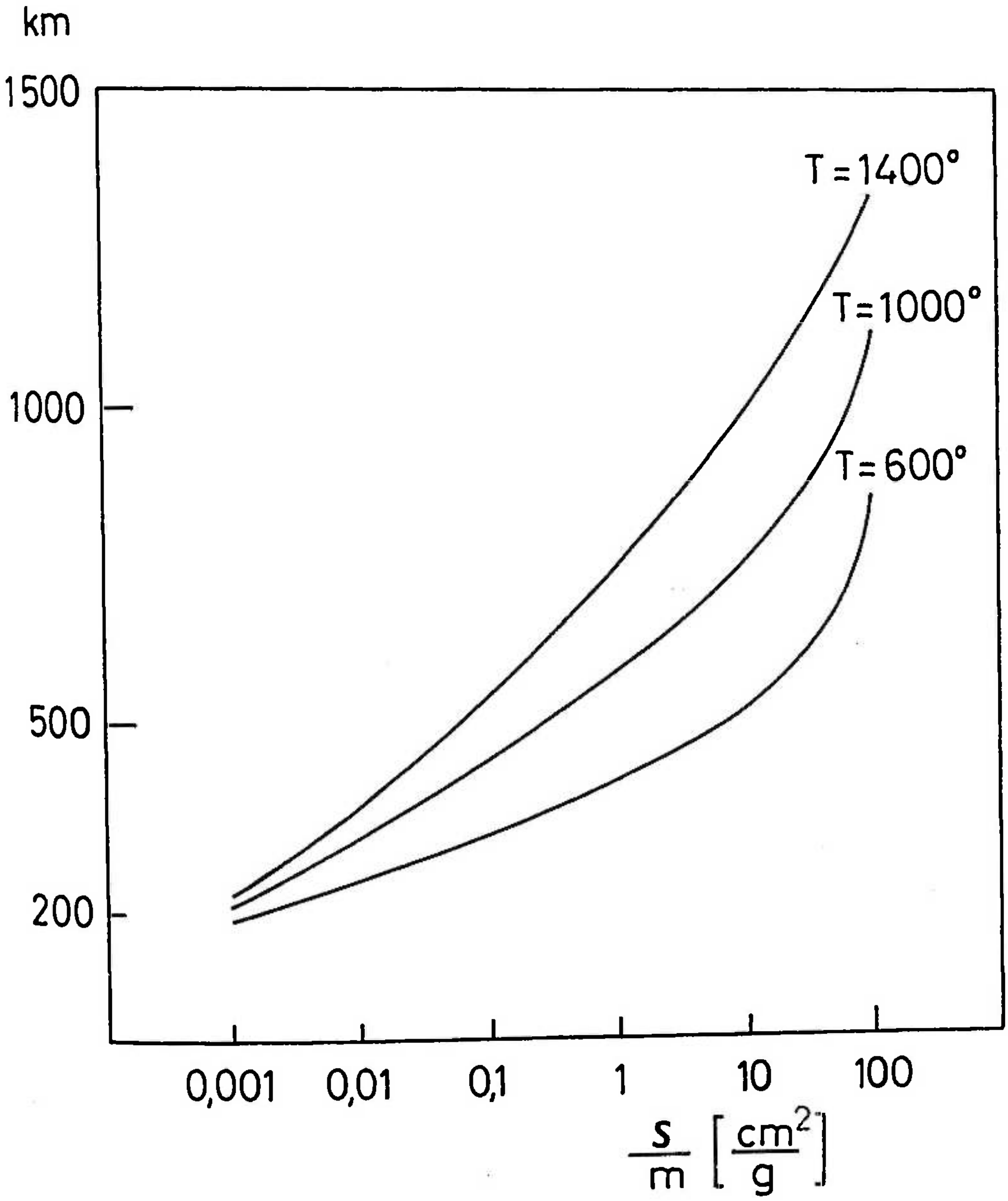

Fig. 6. The heights of equality of the lunisolar perturbations to atmospherical drag, as functions of the S/m ratio, for different exosphere temperatures. Lunisolar perturbations are dominant at heights above the respective curves.

/...

It follows, that the largest values of Δh_p occur if $i' = 0$, $\omega' = \frac{1}{4}\pi\ (1 + 2n)$, where n is an integer. The value of Δh_p depends also on the eccentricity of the orbit, so that the change in perigee height increases with increasing apogee height.

Figure 7 shows the values of maximum changes in h_p, depending on the eccentricity (or apogee height h_a) for two initial perigee heights, 200 and 1 000 km. The lunar and solar disturbing effects are plotted separately.

To compute the long-term changes of the orbital elements, caused by the lunisolar perturbations, we have to admit the time dependence of the position of the disturbing bodies on one hand and of the satellite orbit on the other hand. The motion of the satellite orbit is caused predominantly by the asphericity of the Earth (see part 2), so that the position of the node and perigee are time-dependent quantities.

To get some quantitative insight into the long-term perturbations, let us suppose that the satellite orbital elements are constant during a siderical month (one revolution of the Moon) and let us compute the perturbations during that interval. In the case of the sun we cannot make such an assumption easily since the satellite orbit will be certainly substantially changed during one revolution of the sun, i.e. one year.

Under the above assumption, we get the principal long-periodic perturbations, caused by the lunar attraction. Their period is half the Moon's period of revolution and their amplitudes are given by the expression

$$(\Delta h_p)_{lp} = \frac{15}{16} \frac{\mu_{☽}}{\mu} \left(\frac{a}{r_{☽}}\right)^3 a\, e \sqrt{1-e^2}\, \frac{T_{☽}}{T} \cos i \qquad (12)$$

The maximum change of the perigee height during one lunar month is then given as twice the amplitude.

Figure 8 shows the values of the maximum amplitudes of long-periodic changes, as functions of the orbital eccentricity e for two heights, 200 and 1 000 km.

From the results shown above, we can conclude that the magnitude of lunisolar perturbations does not depend too much on the perigee height but increases strongly with the eccentricity of the orbit (or with the apogee height).

A real example of satellite orbits with extremely high eccentricities is shown in figure 9 which gives the variation of perigee height for satellite Molniya IM (ref. 4) and figure 10 showing the variations of perigee height during the last days of satellite Molniya IS (ref. 5). We can see the rapid decrease in perigee height, caused by the lunisolar perturbations.

These are probably the lowest atmospherical depths into which artificial satellites of the Earth have penetrated; it is with good precision the height of 90 km.

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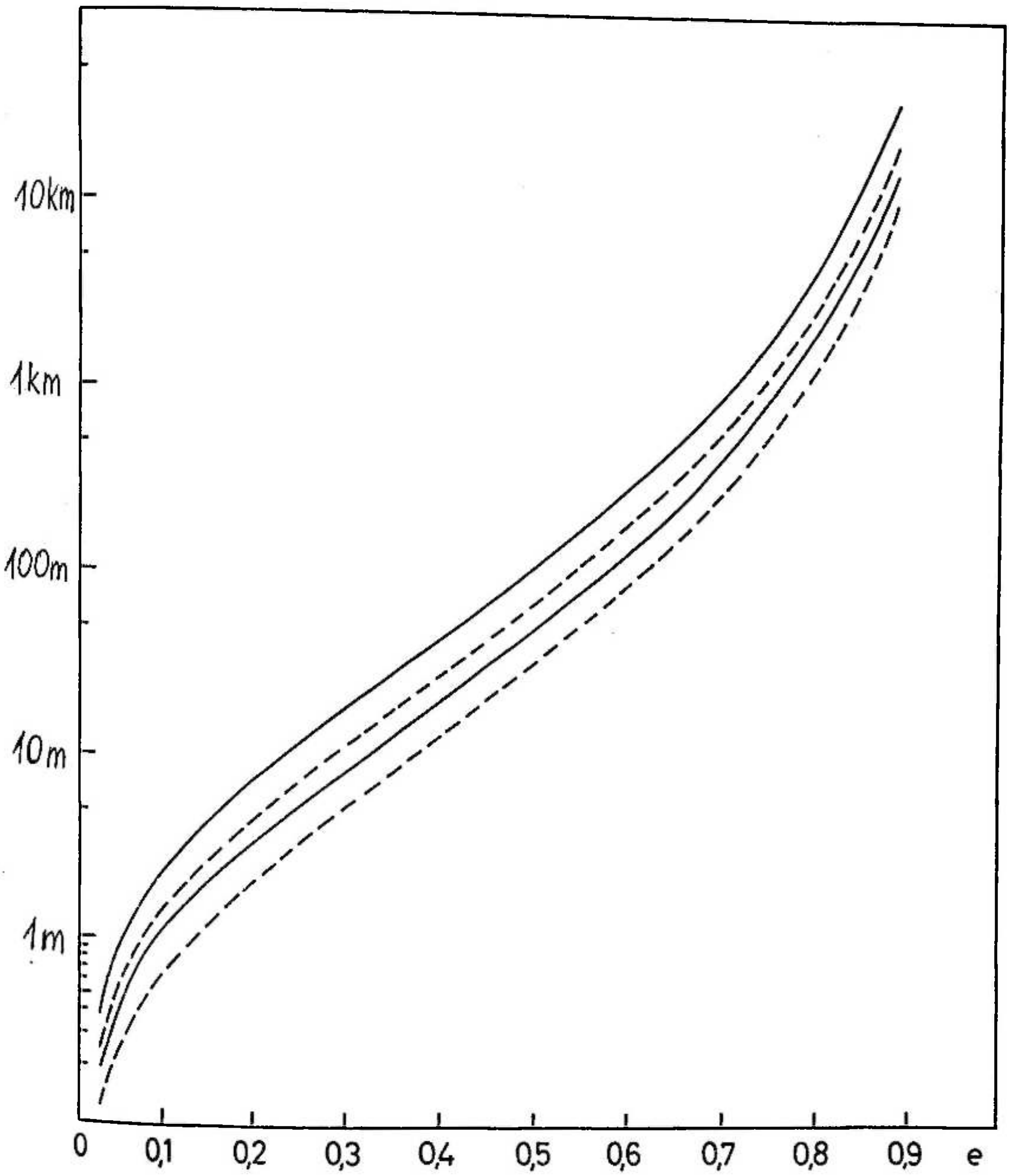

Fig. 7. Lunar and solar perturbations of perigee height during one revolution of a satellite, in dependence on the eccentricity. Full curves correspond to perigee height of 1 000 km, the dotted curves to perigee height of 200 km. Upper curves correspond to lunar, lower to solar perturbations.

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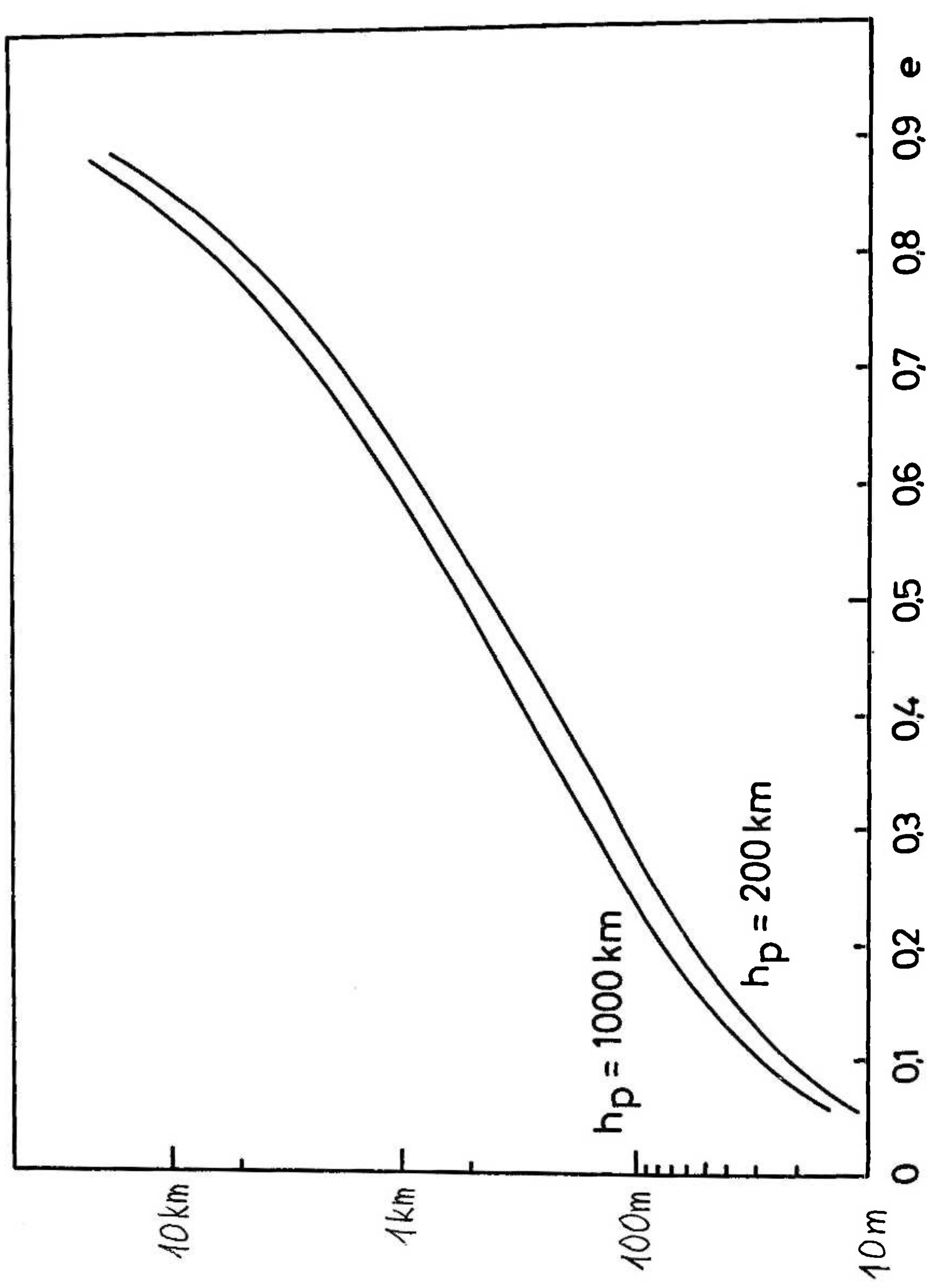

Fig. 8. Maximum values of lunar long-periodic perturbations of perigee height in terms of eccentricity, for two perigee heights.

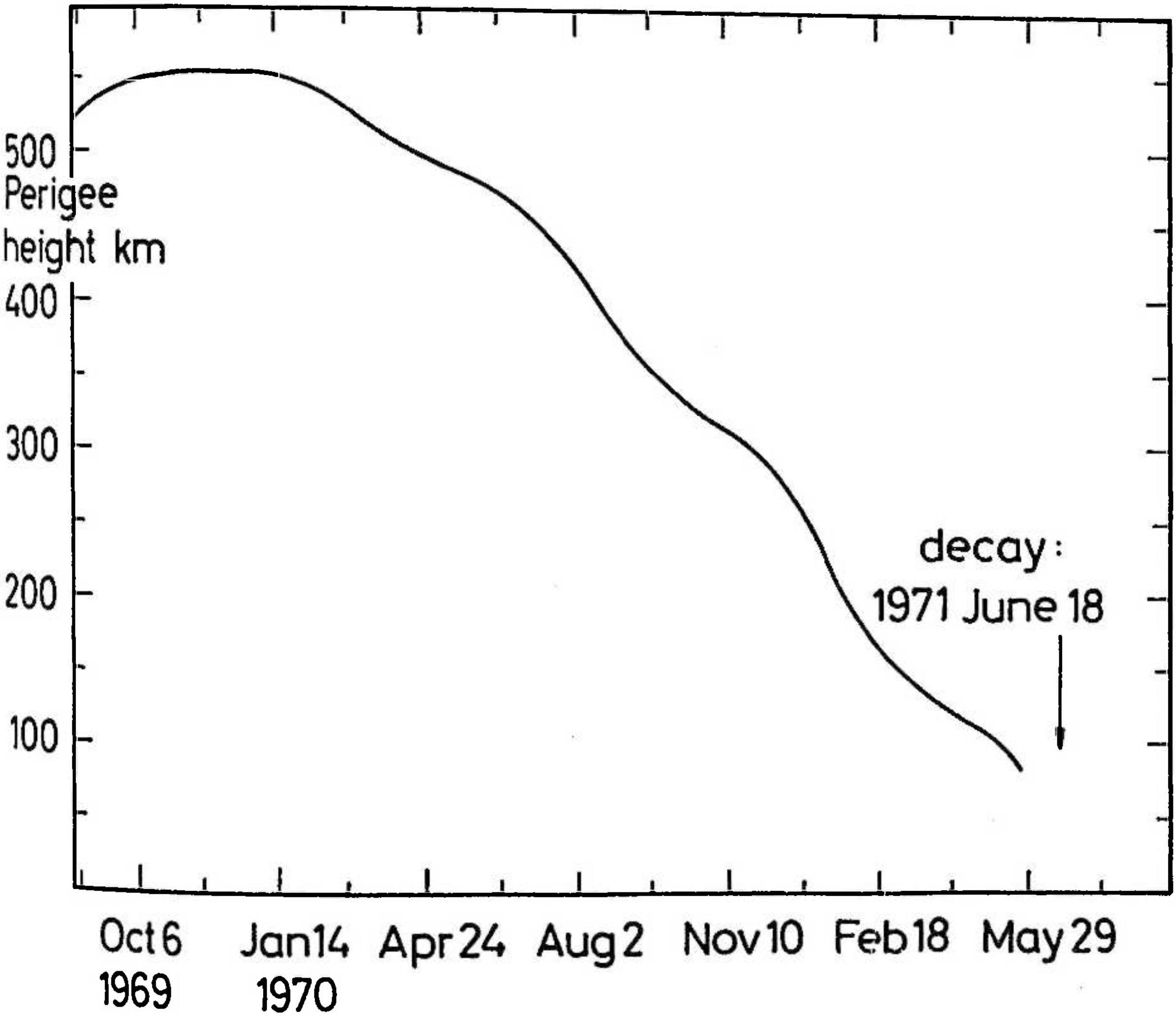

Fig. 9. Variation of perigee height of Molniya IM 1969-61A satellite (Ref. Part 3,/4/).

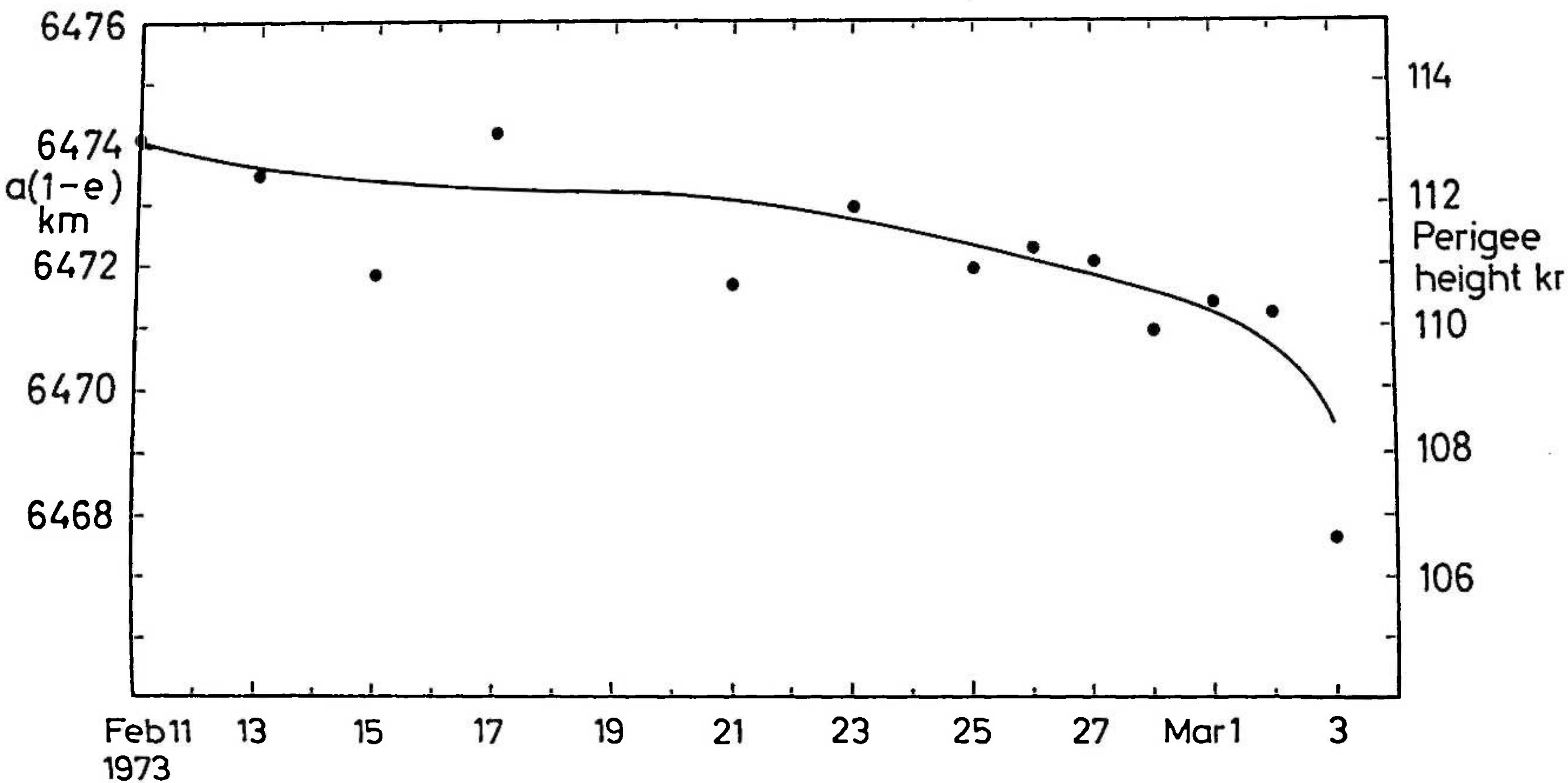

Fig. 10. Values of radius-vector or perigee and of perigee height of Molniya IS, 1970-114F, satellite. The dots are computed orbital values; the curve shows the variation due to atmospheric and lunisolar perturbations (Ref. Part 3,/5/).

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Part 4

Solar radiation pressure and other minor effects

I. The solar radiation pressure effects

The force, exerted on an artificial satellite by the solar radiation pressure depends - as in the case of the atmospherical drag - on the effective cross-section area of the satellite body. Since the gravitational force depends on the mass of the satellite, the solar radiation pressure effects are more pronounced when the ratio of the satellite's mass m to its effective area S is smaller. Therefore, these effects are very important in the motion of balloon satellites (Echo, Pageos) and of small tiny bodies (West Ford needles).

Understandably, the last moments of the satellite's existence, when penetrating the Earth's atmosphere, are determined by the atmospherical drag. However, the solar radiation pressure may alter the orbital perigee height substantially, so that the satellite's lifetime is affected.

The disturbing acceleration F can be expressed as

$$F = \frac{\phi}{c} K \frac{S}{m} \qquad (13)$$

where

ϕ is the solar flux,

c the velocity of light,

K the coefficient of reflectivity (of the satellite's surface),

S the satellite's effective cross-section area, and

m the satellite's mass.

Here we use the ratio S/m which is the inverse of m/S, as used in part 1, since the effects are proportional to that ratio.

Some assumptions are usually made:

1. The satellite is of spherical shape, so that the disturbing acceleration is always in the direction of the sun-Earth line.

2. The solar flux is constant (the variation of the distance of the satellite from the sun is neglected).

[illegible] reflectivity is a constant.

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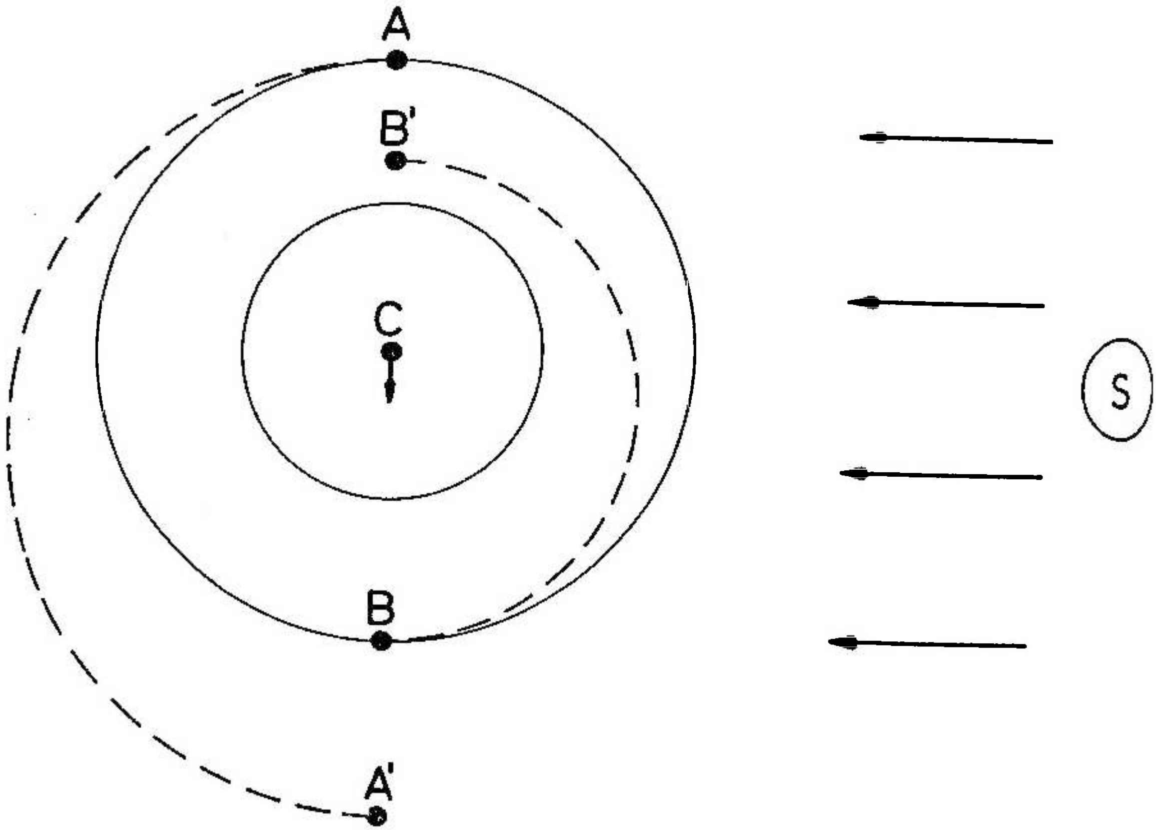

Fig. 11. The solar radiation pressure effect on a circular orbit.

Under those assumptions the solar radiation pressure causes an acceleration in the Earth-sun direction of the magnitude (in cgs units)

$$F_{radp} = 4.65 \times 10^{-5} \times \frac{S}{m} \tag{14}$$

The atmospherical drag depends also on the S/m ratio, according to equation (9) in Part 3.

From these formulae we can compute the height above which the solar radiation pressure effects exceed those of the drag. The height depends on the value of the density of the atmosphere, which changes according to the solar activity. Thus, the corresponding heights are computed from the equation

$$\frac{\rho}{r} = 2 \; \frac{K\, q.}{C_D\, \mu} \tag{15}$$

where μ is the geocentric gravitational constant and q. is the solar radiation pressure exerted on a unit surface. The resulting heights exceed 600 km.

Let us first visualize roughly the simplest case of solar radiation pressure effects. If a satellite moves (Fig. 11) in a circular orbit the plane of which contains the Earth-sun line, then the satellite is accelerated on its path away from the sun (from point A) and instead of reaching point B, is pushed towards A'. In the other half of its orbit, the satellite tends to go to the point B'. As a result, the eccentricity of the orbit increases.

There is one circumstance, that causes difficulties when treating that effect. This is the effect of the Earth's shadow, when the solar radiation pressure ceases to act. This discontinuous effect disrupts the symmetry in gaining and losing energy of the satellite through the radiation pressure, and as a consequence secular changes of the semi-major axis arise.

Let us now consider the changes of the perigee height neglecting the shadowing effect. We can express that change, during one revolution of the satellite, as

$$\Delta h_p = -3\pi\, K\, q.\, \frac{S}{m}\, \frac{a^2}{\mu}\, a \sqrt{1 - e^2} \cos i' \sin\omega' \tag{16}$$

where i' and ω' depend on the position of the orbital plane with respect to the direction to the sun. This expression gives the maximum changes of the height of the perigee depending on S/m and on the eccentricity. This effect is important at much higher altitudes than those considered here.

During long time intervals the changing orbital elements and also the changing position of the sun have to be taken into account. Such long-periodic changes are particularly important in cases of resonance. However, also these effects appear in orbits with very high perigee altitudes.

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Since the position of the orbit in space is affected by the time when the satellite is injected into its orbit, the lifetime of the satellite is affected by the time of the launch, too. The ratio of the lifetimes of satellites launched into the same orbits but at different day-times, can reach the factor of 10.

This fact can be used to select the lifetime of a satellite (having high S/m ratio) by a proper choice of the initial launching date and of the resonant conditions. This was done, e.g., in the case of the project "West Ford" - orbiting needles for radio-communication. The orbit of the needles was chosen so as to remove them from orbit in about three years after launch.

II. Other minor effects

There are some other effects which must be taken into account when studying the dynamics of an Earth artificial satellite motion in detail. All of them may have some influence on the perigee height; however, those effects are small. Let us quote some of them:

1. Charged drag:

   The accretion of ions and electrons may change the satellite's effective cross-section area.

2. The solar radiation reflected (diffusely or specularly) from the Earth's surface.

3. The Lorentz force:

   An electrically charged satellite moves in the magnetic field of the Earth.

4. The Poynting-Robertson effect:

   The action of radiation, arising as a result of non-vanishing velocity of the satellite.

5. The Earth tidal effects:

   The changing gravitational field of the Earth caused by tides in the solid Earth.

Part 5

The initial and final stages of satellite's orbit

I. Injection into orbit

The aim of a launcher rocket is to transport the maximum payload into the orbit. I.e., the rocket has to work with maximum effectiveness, having minimum energetic losses during the ascent. Besides the technical design, an important role is played here by the proper choice of the ascent trajectory.

The change of the rocket velocity V is described by the fundamental equation of rocket propulsion:

$$m \frac{dV}{dt} = p - X - m\, g \sin \theta \quad , \qquad (17)$$

where m is the mass of the rocket, g is the gravitational acceleration, θ is the angle between the velocity vector and the horizontal plane, and P is the thrust of the rocket. The thrust depends on the exhaust velocity w and on the velocity of the fuel consumption (dm/dt):

$$P = - w \frac{dm}{dt} . \qquad (18)$$

There are three terms on the right side of equation (17) - the first expresses the active propulsion, the other two concern energetic losses: X denotes aerodynamic losses and $m\, g \sin \theta$ expresses the effects of the Earth's gravitation.

The losses during the flight of a typical rocket are shown in figure 12 and numerical values are given in table 4 as an example for the rocket Saturn 5.

To diminish aerodynamic losses, the rocket has to be launched vertically upwards with the initial acceleration as low as possible (the minimum is 1.25 g). Saturn 5 starts with the acceleration of 1.3 g; the atmospherical drag is at its maximum value at 12.8 km, 76 sec after launch, and the acceleration reaches its maximum of 3.9 g after 150 sec, at 65 km altitude.

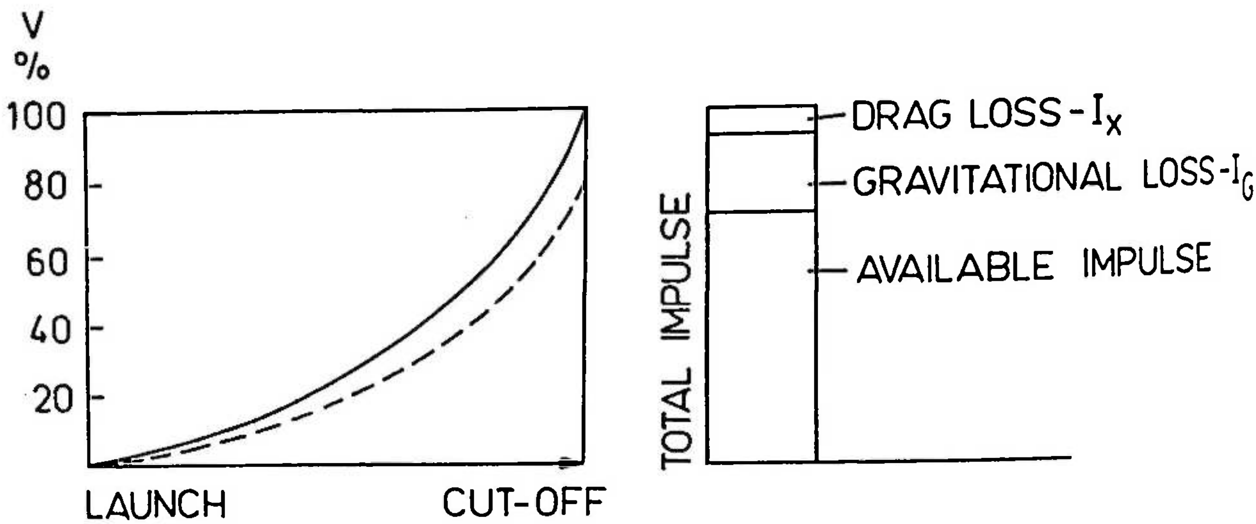

Fig. 12 Gravitational and aerodynamic losses for a typical launcher rocket. On the left side in terms of total vehicle velocity v, on the right side in terms of total rocket impulse.

Table 4

Gravitational and aerodynamic losses during ascent of rocket Saturn 5.

Stage	Theoretical velocity m/sec	Gravitational losses m/sec	Aerodynamical losses m/sec
1	3 660	1 220	46
2	4 725	335	0
3	4 120	122	0
Sum	12 505	1 677 (13.5 %)	46 (0.4 %)

The gravitational losses are more important than the aerodynamical losses. To diminish them, the path of the rocket is bent as early as possible after a vertical start, and the ascent is accomplished in a direction parallel with the Earth's surface. The gravitational losses would be more than doubled in a fully vertical start.

After the powered ascent, the payload has to be given the necessary orbital velocity. Orbital velocity is usually attained at the lowest point of the orbit. If larger distances are to be reached, or the flight to other celestial bodies is planned, the satellite is first put into a "parking orbit" round the Earth, and the final impulse is imparted later. The minimum altitude of a parking orbit is determined by the atmospherical drag effects (see part 1).

The lowest altitude used up till now was about 158 km (project Gemini). The space shuttle will be injected into orbit at about 120 km (120 sec after start); however, this height will be increased half a revolution later to about 180 km to reach an orbit with a sufficient lifetime.

Summing up, the following conditions should be fulfilled during the ascent into orbit:

1. Vertical start,

2. Horizontal flight at the final phase of the ascent,

3. Injection height above 100 km.

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The choice of the orbit is then the task of the optimization of the rocket trajectory.

In the early age of astronautics it was not possible to follow always the optimum trajectory. Thus, the rockets reached the orbit relatively fast and closer to the launching site. Also, simple rockets powered by a solid fuel are not injected into orbits using optimum trajectories. E.g. the Japanese rocket L-4 S starts at the angle of 63° towards the horizon and is injected into the orbit at the height of 360 km in a surface distance (surface downrange) of 1,400 km from the launching site.

Large modern rockets for special astronautical purposes are usually launched in optimal trajectories with very similar characteristics. A few examples are given in table 5. A two-stage rocket reaches the orbit at a distance of 1,800 km from the launching site and a three-stage rocket at a downrange of 2,600 km. An example is illustrated in figure 13.

It is a question if the duration of ascent and the downrange distance could become shorter in the future. One possibility could be the "fully reusable space shuttle", a one-stage rocket (SSTO - Single Stage to Orbit), with no parts jettisoned during the ascent.

Impact of jettisoned rocket stages

At the present level of rocket technology, it is impossible to transport a payload into a satellite orbit using a single-stage rocket. This means, that during the ascent, one or two empty rocket stages have to be jettisoned. When not destroyed in the atmosphere, they fall onto the Earth's surface. The distance of the impact from the launch site can be larger than that in which the injection into orbit takes place because the jettisoned parts of a rocket follow a ballistic trajectory but the location of the expected impact can be predicted. E.g., during the Apollo missions, the impact of the first and second rocket stages occurred at distances of 665 km and 4,048 km from the launching site. The two outburned boosters of the space shuttle are expected to fall down at a distance of 300 km and the empty fuel tank at more than 5,000 km from the launch site.

The deviation from the planned direction of the ascent trajectory does not exceed 100 km, which at 5,000 km subtends an angle of 0.6 degrees.

II. Atmospheric entry

The descent from the satellite geocentric orbit is achieved by diminishing the orbital velocity. The Earth's atmosphere is sufficiently dense to provide a braking effect so that it is not necessary to use a special rocket engine. Such engine is used only to direct the satellite into the proper entry corridor.

Table 5

The principal characteristics of the ascent of different rockets

Rocket	Time of ascent (sec)	Down range (km)	Height (km)
Soyuz	530	1 600	185
Saturn 1	590	1 830	158
Saturn 5 (two-stage)	590	1 820	442
Atlas Centaur	640	2 400	226
Saturn 5 (three-stage)	684	2 570	191
Space Shuttle	700	2 600	120

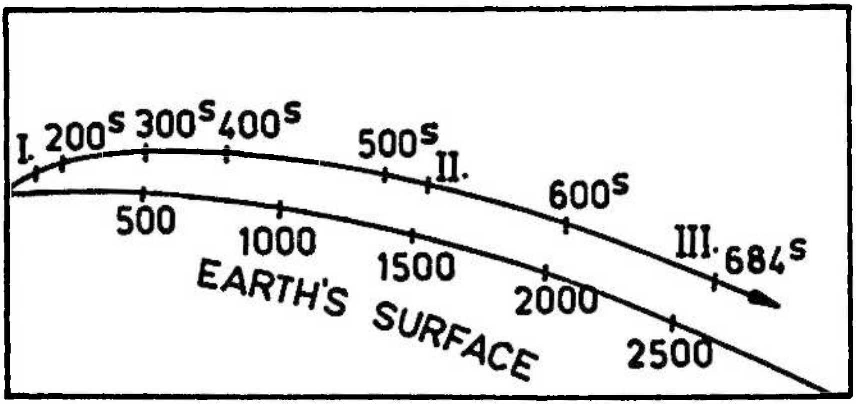

Fig. 13 Ascent trajectory of the Saturn V rocket. Time is in seconds, surface range in km. Points of individual rocket stages shut-down are labeled by corresponding roman numerals.

The concept of a reference effective atmospheric boundary (so-called "entry interface") is used to describe the conditions of atmospheric entry. It is usually put at the height of 122 km (400 kft) sometimes also 111 km (60 nm). The entry interface of 100 km is used in the USSR. The atmospheric entry is defined as a passage through the entry interface and the motion at lower heights is considered as a motion in the atmosphere. However, it is only an accepted usage. The angle θ is another parameter for the description of the atmospheric entry - it is the deviation from the local horizontal plane; see figure 14.

The descent with a zero lift is called "ballistic re-entry". The satellite does not perform any manoeuvring, so that the shape of the descent trajectory is determined by the conditions of re-entry only.

The maximum admissible deceleration is a limiting factor for the angle of entry θ, which again determines the down range. The values of deceleration are shown in table 6.

The deceleration increases rapidly with the angle θ and it can achieve inadmissible values for the safety of manned space crafts. The physiologically admissible deceleration is 8g for 4 minutes, 12g for 1 minute and 16g for 10 seconds. The re-entry along a ballistic trajectory can be made from a low orbit under an angle θ of about 3°. The dependence of the down range on the entry angle is shown in figure 15 and the corresponding deceleration in figure 16. The whole descent takes 200-800 seconds.

The conditions at the return from the moon (or from an interplanetary flight) are shown in figures 17 and 18. If the angle θ is lower than 3°, then a "total atmospherical skip" occurs and the satellite bounces off the atmosphere.

Figures 17 and 18, and table 6, indicated that the surface down range is relatively short but such orbits are not suitable for manned missions. They have, however, the advantage of not requiring an on-board control system.

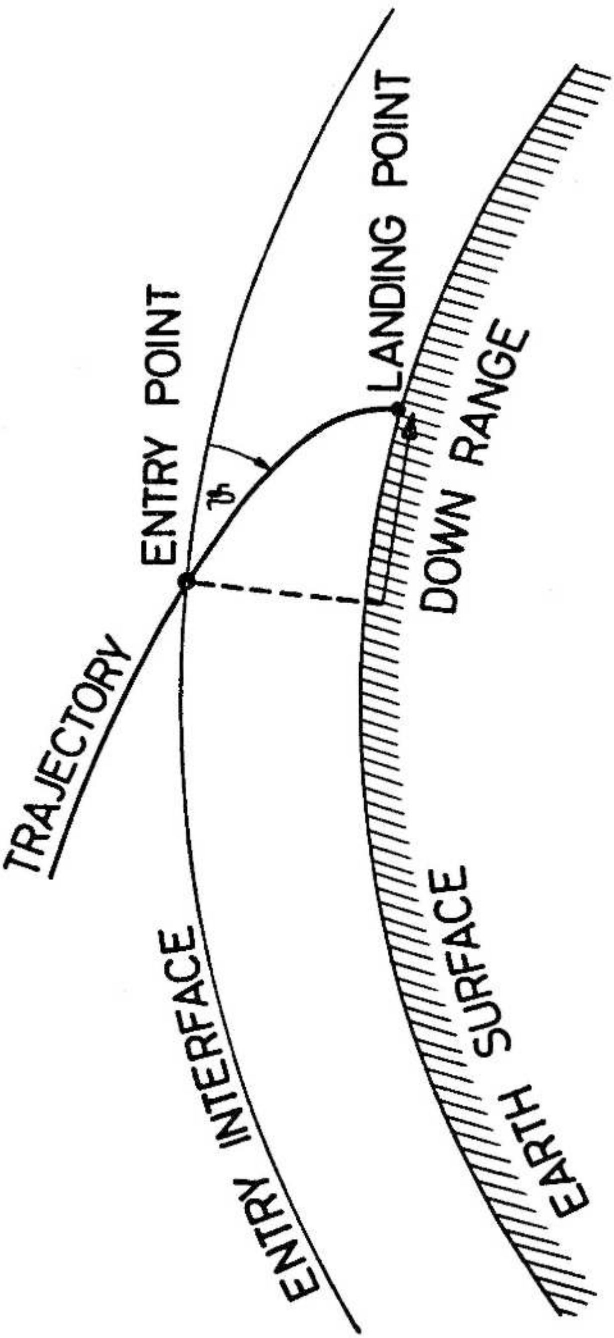

Fig. 14. Geometry of the atmospheric re-entry

Table 6

The maximum deceleration (in units of surface gravity acceleration g) during descent, for different velocities (in units of circular velocity V_C = 7.83 km/sec at a height of 120 km), in terms of the angle ϑ .

ϑ \ V/V_C	1.0	1.3	1.4	1.5
3°	11 g	15 g	-	-
6°	18	28	16	-
10°	30	46	53	61
30°	53	132	153	175
65°	128	239	278	317
90°	156	264	305	350

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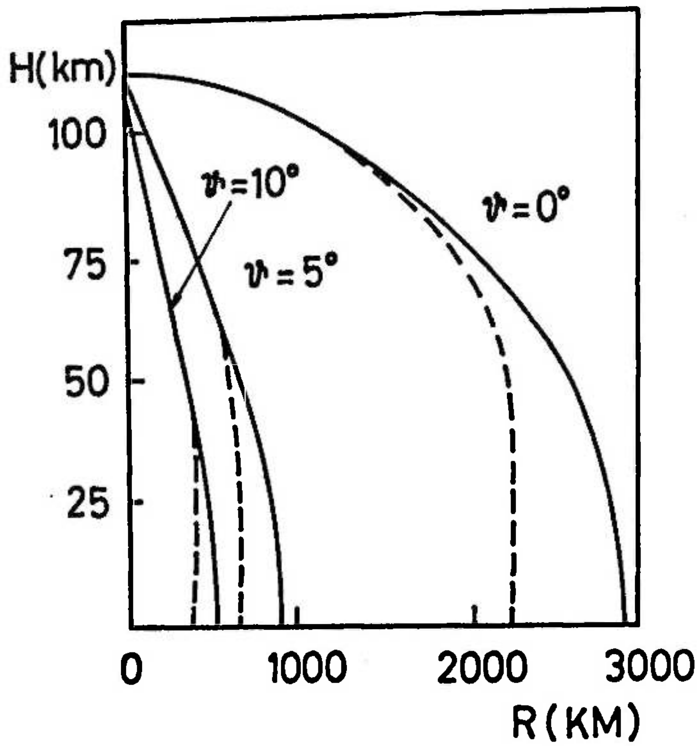

Fig. 15. Re-entry paths from a low orbit for two typical area/mass ratios (full and broken lines). The height H is shown as a function of the down range R.

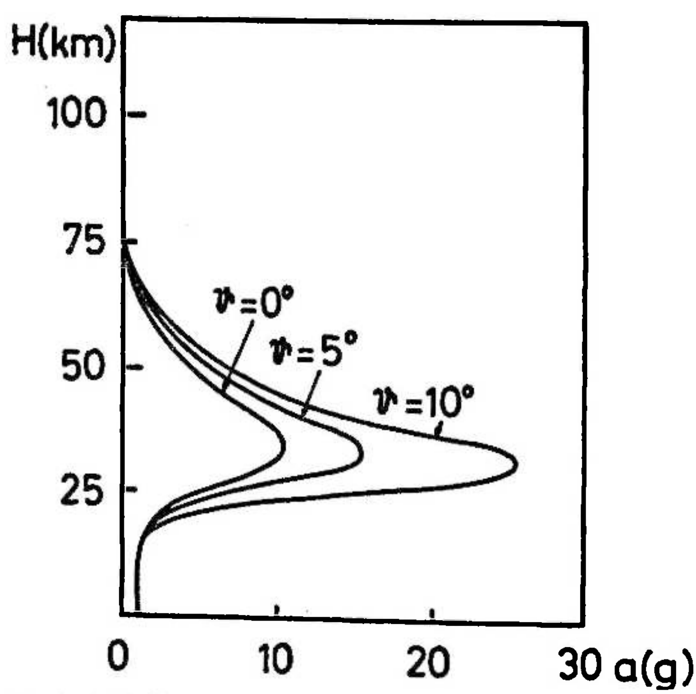

Fig. 16. The height H and deceleration a of a satellite for various values of the re-entry angle ϑ .

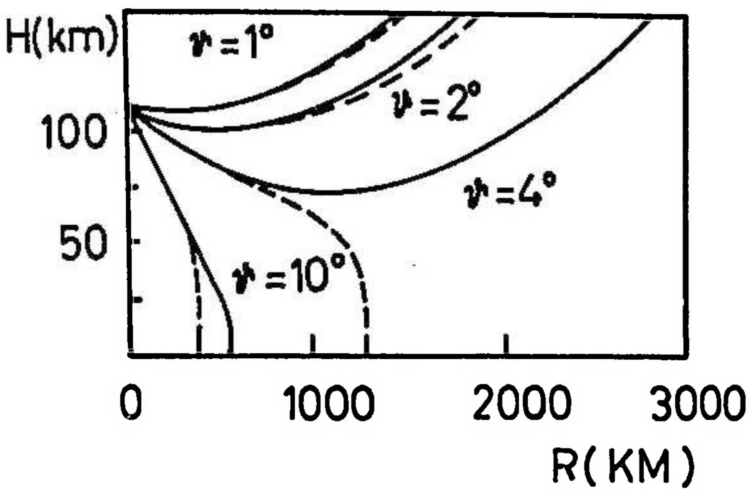

Fig. 17. Re-entry paths from a lunar or interplanetary flight.
The notation is the same as in Fig. 15.

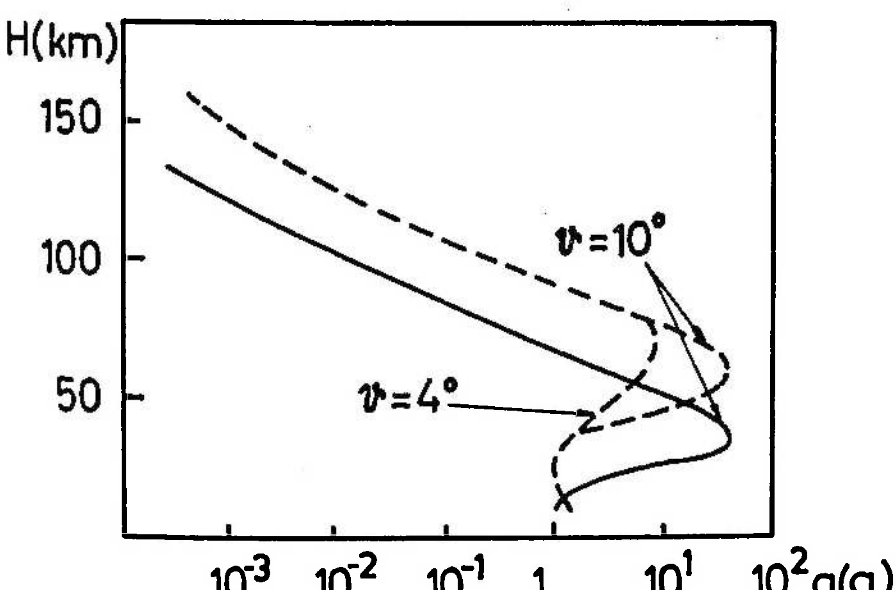

Fig. 18. The height H and deceleration a of a satellite during re-entry from a lunar or interplanetary flight.

A descending body producing aerodynamic lift makes an "aerodynamic re-entry". It allows manoeuvring, the deceleration can be substantially lower, and the down range can be changed. The length and duration is always greater than during a corresponding ballistic descent.

The possibility of manoeuvring extends the admissible entry corridor (the region of values of angle θ) for a given system. The extent of the entry corridor is determined on one side by the admissible deceleration and on the other side by the condition of capture of the body by the atmosphere. For a given deceleration, we have certain limits: e.g. for deceleration of 10g the extent of the corridor is 0.4° in case of ballistic descent, whereas in case of the aerodynamic re-entry the corridor is about 10 times wider.

When returning to the earth from a parabolic trajectory (from a lunar or interplanetary mission), at an angle θ under 5.5°, the body bounces off the atmosphere, decelerates and makes a second final re-entry. As an example, the velocity of the Soviet probe Zond 6, decreased during the first atmospheric entry from 11 km/s to 7.6 km/s; the second entry diminished the velocity to the value of 200 m/s before the parachuting. The probe travelled in the atmosphere a distance of about 9,000 km.

The manoeuvring capabilities of Apollo are shown as an example in figures 19 and 20. The distance to the landing point could have been changed in a range of about 4,500 km and the maximum cross range was almost 600 km.

Descent paths of several systems are shown in figure 21. The planned trajectory of the Space Shuttle is the longest one, even if it is placed in a low orbit. The reason is the limit put on the deceleration not to exceed a value of about 3g.

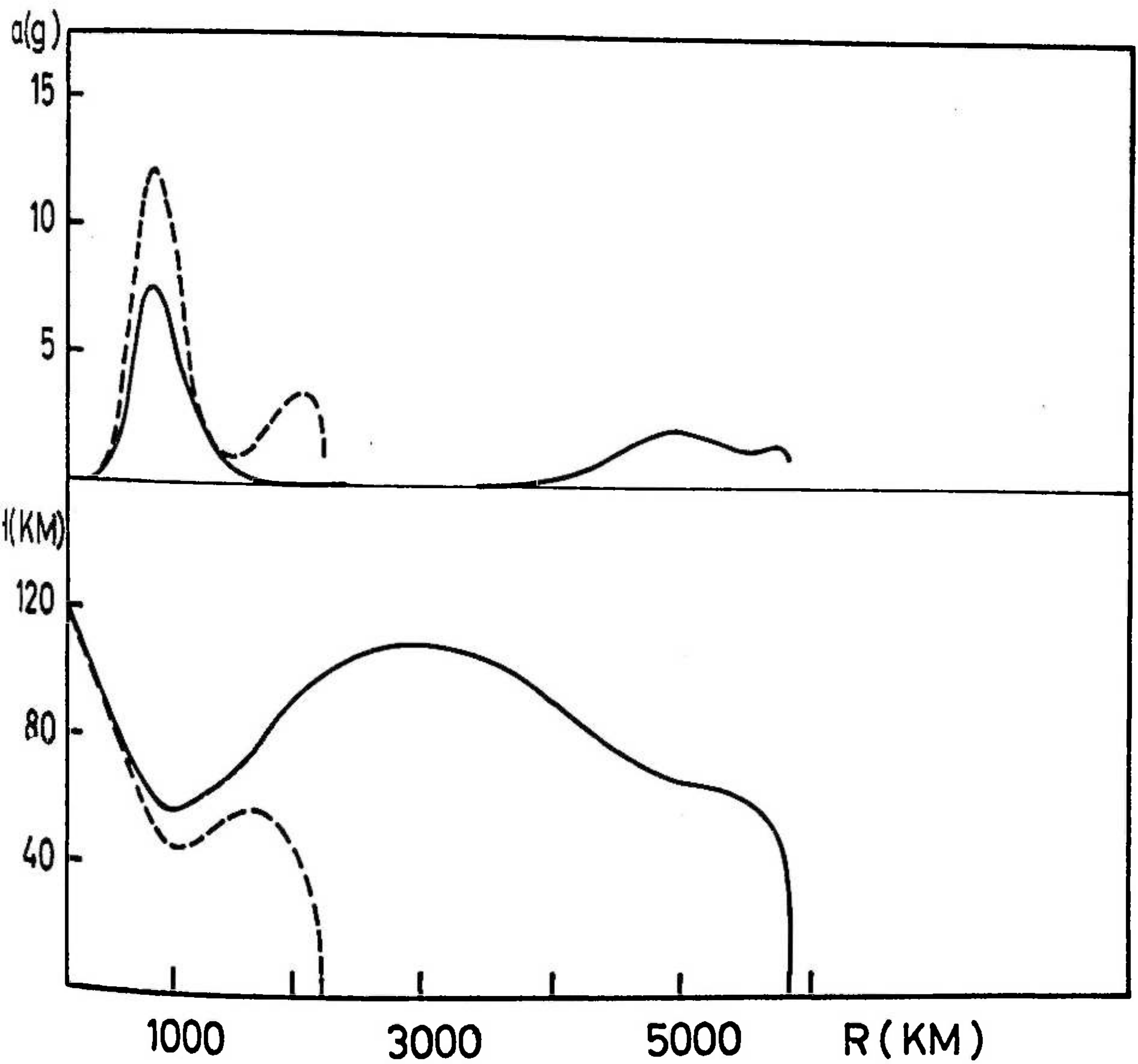

Fig. 19. Two examples of Apollo-type vehicle aerodynamic re-entry for a down range of 2200 km (broken line) and 5700 km (full line). The upper part shows the deceleration a , the lower part the height H.

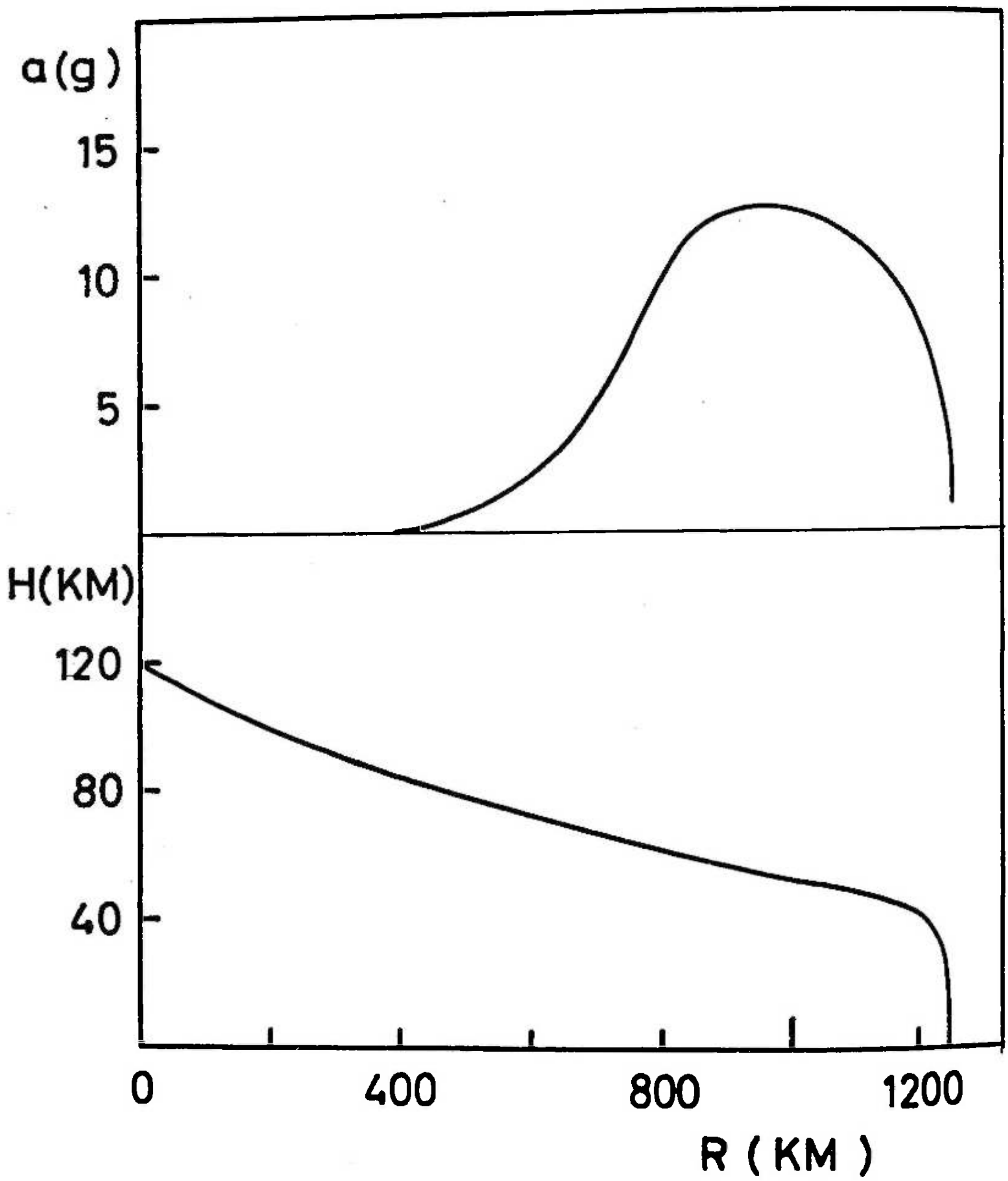

Fig. 20. Another example of the aerodynamic re-entry of an Apollo-type vehicle showing the minimum possible down range R and fulfilling the condition that the deceleration a never exceeds the value of 10 g .

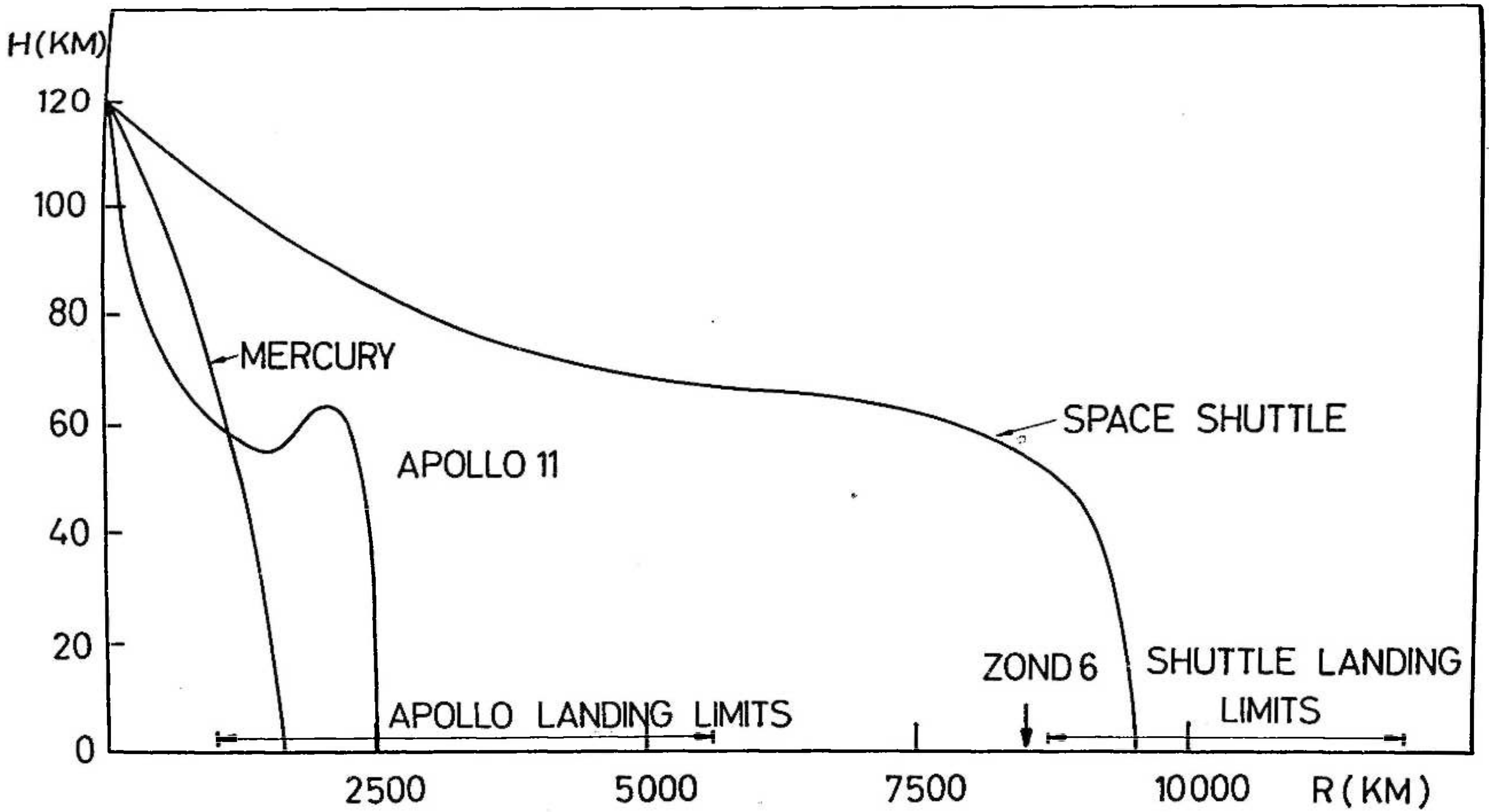

Fig. 21. Re-entry trajectories of several space systems giving the height H against the **down range R. Manoeuvring ranges are** shown for Apollo and Space Shuttle. The detailed trajectory of the Soviet Moon probe Zond 6 was not available.

The manoeuvring capability of the Space Shuttle along its path is not larger than that of Apollo. It has however a large cross range manoeuvring capability of 2,000 km, to make it possible for the pilot to land on a runway 5 km long and 90 m wide with a velocity of 330 km/hour. The planned path of the Space Shuttle descent is described in more detail in table 7 and the final phase of controlled aerodynamical descent is shown in figure 22.

As a conclusion we can say:

The length of the descent trajectory of a body, from the moment of interface entry (at a height of 100-110 km) depends mainly on the admissible value of deceleration. In case of automatic systems the trajectory can be short, but for future flights, and especially for all manned flights, the long descent trajectory will probably be used. This enables a controlled descent with lower values of deceleration. The distance of down range is up to about 10,000 km and the manoeuvrability in cross range is about 2,000 km.

Table 7

Main stages of Space Shuttle landing:

T - time (s) elapsed from the atmospheric interface entry

h - height (km)

v - velocity (km/s)

Flight phase	T (s)	h (km)	v (km/s)	Range (km)	
				down	cross
Interface entry	0	122	7.813	0	0
Deceleration 0.005 g	190	100	7.838	1 700	0
Pitch down	1 525	47	2.438	9 000	1 100
Maximum or dynamical pressure	1 790	27	0.985	9 300	1 700
End of pitch down	1 905	22	0.463	9 400	1 800
Entry into interface boundary (figure 28)	1 909	21	0.454	9 500	2 000

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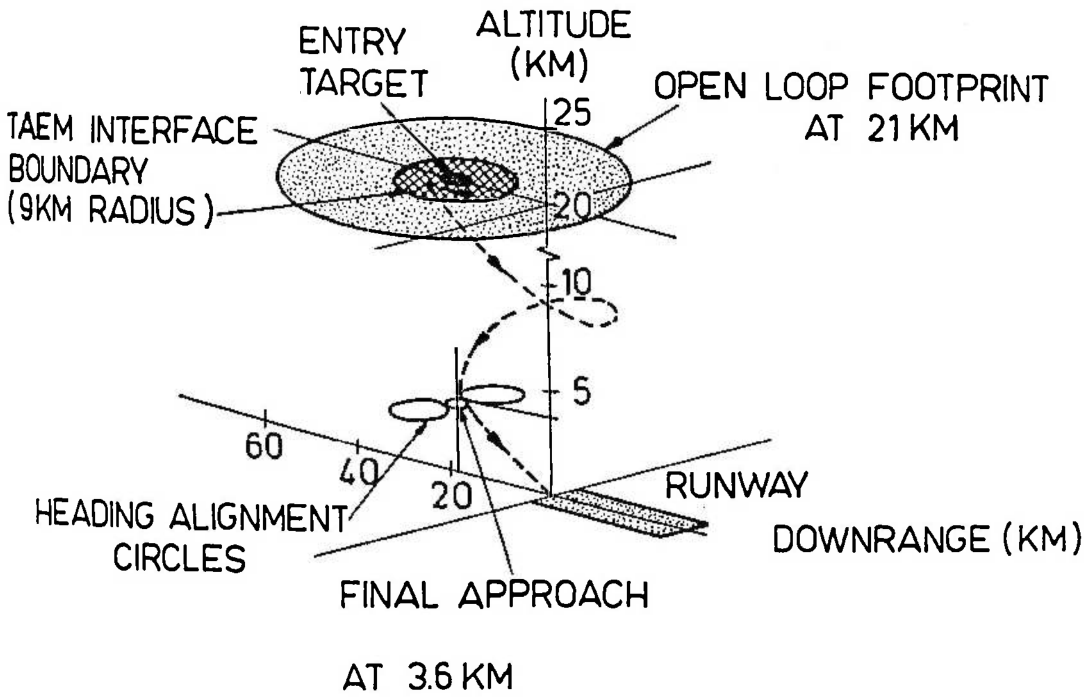

Fig. 22. Final phases of the Space Shuttle descent (for the beginning of re-entry see Fig. 27). The pilot must reach the interface boundary with a radius of 9 km at a height of 21 km. Following the controlled, but passive (without engines), aerodynamic descent, the Shuttle goes through radio heading alignment circles at 3.6 km to final approach to the runway.

List of symbols

a - the semi-major axis of satellite orbit

c - velocity of light

C_D - aerodynamic drag coefficient

e - the eccentricity of satellite orbit

g - gravitational acceleration

h - height above the surface of the Earth

h_p - perigee height

Δh_p - change of perigee height

H - atmospheric density scale height

i - the orbital inclination

i' - orbital inclination if the reference frame of disturbing body

J_i - harmonic coefficients in the definition of the gravitational field (of order i)

K - coefficient of reflectivity of satellite's surface

L - lifetime of a satellite

m - mass of a satellite

m_D - mass of the disturbing body

$m_\oplus$ - mass of the Earth

$m_{☽}$ - mass of the Moon

$m_\odot$ - mass of the Sun

q_o - solar radiation pressure exerted on a unit surface at a distance of the Earth from the Sun

$R_\oplus$ - radius of the Earth

$\bar{r}$ - radius-vector of a satellite

/...

r - length of the radius-vector

$\bar{r}_D$ - radius-vector of the disturbing body

r_D - length of radius-vector $\bar{r}_D$

$r_{☽}$ - radius of the Moon's geocentric orbit

$r_{\odot}$ - radius of the Sun's geocentric orbit

r_S - radius of a spherical satellite

S - cross-sectional area of a satellite

T - orbital period of a satellite

$\dot{T}$ - rate of change of orbital period T

T_o - time of perigee passage of a satellite

$T_{☽}$ - period of revolution of the Moon

V - orbital velocity of a satellite

V_C - orbital velocity of a satellite on a circular

w - exhaust velocity of burned fuels of a rocket engine

X - aerodynamic losses during ascent

<u>Greek symbols</u>

ϕ - solar flux

θ - angle between the trajectory of descending satellite and local horizontal plane

μ - geocentric gravity constant

μ_D - seleno- or helicentric gravity constant

$\rho, \rho A$ - the density of the atmosphere

$\rho(h)$ - the density of the atmosphere at the height h

ω - the argument of perigee of satellite orbit

ω' - the argument of perigee in a reference frame of a disturbing body

Ω - the longitude of ascending mode of satellite orbit

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Part 1

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Part 2

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Annex II

Statistics of lowest altitudes of satellites launched between 4 October 1957 and 4 November 1975

The number of launchings of artificial satellites of the earth in the above time interval is 1537. A few of these were space probes which after a short time in orbit left the neighbourhood of the earth. Some were satellites put into high orbits where a long lifetime was needed to accomplish planned missions. Some will orbit the earth for years, decades, centuries and even longer. About one third of all the launches are still in orbit. Other satellites or rockets which were placed into low, nearly circular, orbits or eccentric orbits with a low perigee height had a short lifetime of some weeks or days. A few of these satellites were brought back to earth but the majority burned up in the atmosphere.

Satellites are being regularly observed and their orbits computed by several institutions. A very detailed list of orbits is computed, compiled from other sources, and published by the Royal Aircraft Establishment, Farnborough, Hants, England, in the Table of Earth Satellites, vol. 1 (1957-1968), vol. 2 (1969-1973), vol. 3 (1974 onwards).

Satellites with lowest perigee heights have been excerpted from the above catalogue and plotted in a diagram (Figure 1) in dependence of the eccentricity of their orbits. The diagram is complete below the line indicated and its left part has been drawn in an expanded scale. Each satellite is represented by a dot. The number of days elapsed between the determination of its orbit and its decay is indicated by appended numbers. Unnumbered dots refer to satellites which decayed within one day and asterisks refer to satellites which stayed in orbit for 10 or more days.

The diagram shows that satellites in nearly circular orbits live at low altitudes for a very short time only. Satellites in eccentric orbits can survive close approaches to the earth for a longer time. More details on satellites which lived for a certain period of time at or below 120 km are given in table 1.

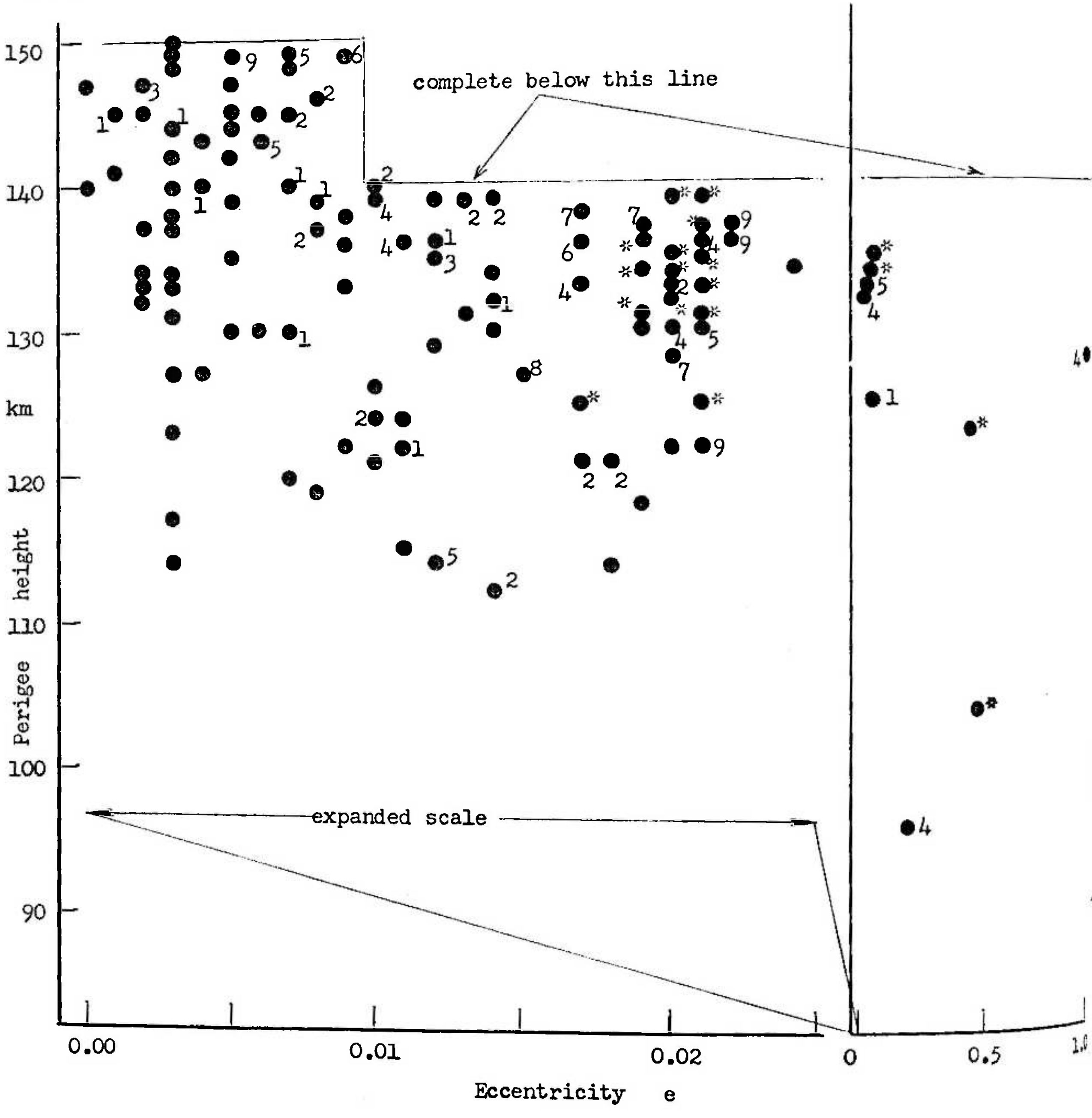
complete below this line
expanded scale
Perigee height km
150
140
130
120
110
100
90
0.00
0.01
0.02
0
0.5
Eccentricity e

Figure 1

Name		Launch date Lifetime Descent date	Date of orbital determination	Perigee height (km)	Apogee (km)	Orbital eccentricity
1970-56B	Cosmos 354 rocket	1970 Jul 28.92 0.36 days 1970 Jul 29.28	1970 Jul 29.2	114	157	0.003
1970-76C	Cosmos 365 rocket	1970 Sep 25.59 0.3 days ? 1970 Sep 25	1970 Sep 25.7	117	161	0.003
1967-91B	Cosmos 179 launch platform	1967 Sep 22.59 0.43 days 1967 Sep 23.02	1967 Sep 22.8	120	212	0.007
1969-108A	Cosmos 316	1969 Dec 23.39 248.46 days 1970 Aug 28.85	1970 Aug 28.0 Pieces of satellite picked up on earth after re-entry	119	226	0.008
1966-12B	Bluebell 2 cylinder	1966 Feb 15.85 0.72 days 1966 Feb 16.57	1966 Feb 16.3	115	253	0.011
1966-101G	Cosmos Capsule	1966 Nov 17 185 days 1967 May 6	1967 May 1.1	114	272	0.012
1971-68B	Cosmos 433 launch platform	1971 Aug 8.99 1.04 days 1971 Aug 10.78	1971 Aug 9.2	112	300	0.014
1974-103B	Cosmos 699 rocket	1974 Dec 24.64 1 day 1974 Dec 25	1974 Dec 24.7	114	358	0.018
1975-102B	Cosmos 777 rocket	1975 Oct 29.46 1 day 1975 Oct 30	1975 Oct 29.7	118	373	0.019
1974-02A	Skynet 2A	1974 Jan 19.07 6 days 1974 Jan 25	1974 Jan 20.7	96	3406	0.204
1966-92D	Molniya 1D rocket	1966 Oct 20.33 959.06 days 1969 Jun 5.39	1969 Feb 16.0	104	11321	0.464

Table 1. Satellites with perigee height equal to, or less than, 120 km published in the Table of Earth Satellites 1957-1975 Nov 4.
